Revisiting the Supply-Side Effects of Public Debt in a Banking Model

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First Version:October 2019
This Version: July 2021

Abstract

I build a tractable real business cycle model with financial frictions in the banking sector. A key assumption is that Treasury securities are the major collateral in the wholesale interbank market. I show that Ricardian equivalence fails if the supply of public debt is lower than a certain threshold. A shortage of public debt will reduce credit supply and encourage financial speculation at the same time, with the two channels having different effects on capital accumulation. Given a mild shortage of public debt, the net impact is that credit spread arises, asset price dispersion increases, and aggregate output declines. A permanent increase in government spending, if financed by public debt, has positive externalities stemming from the improvement of financial intermediation. In a more general model calibrated to the U.S. economy, I find that a small negative investment opportunity shock can generate a sizable economic recession. A debt-financing government spending can generate a cumulative fiscal multiplier above one, with both investment and consumption being crowded in.

Keywords: Public debt, fiscal multiplier, financial constraints, financial intermediation.

JEL Classifications: E13, E22, E32, E44, E47, E62, G20.

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1 Introduction

Whether or not Ricardian equivalence holds has far-reaching implications for monetary and fiscal analysis. A large literature emerges after Barro (1974) 's seminal contribution, continually refining our understanding of both its empirical and theoretical importance. This paper constitutes one attempt to explore the possible non-neutral impacts of public debt on financial intermediations and the size of the fiscal multiplier.

The question is especially relevant in the context of the Great Recession, in which the dry-ups and runs in the wholesale banking market play an important role. In this market, the repurchase (repo) contract is a major instrument used by financial institutions to borrow against each other.[1] Roughly two-thirds of collateral used in the 5-trillion-dollar repo market are Treasury securities. This suggests that the supply of Treasury securities can have a non-trivial influence on the liquidity conditions of the financial market.

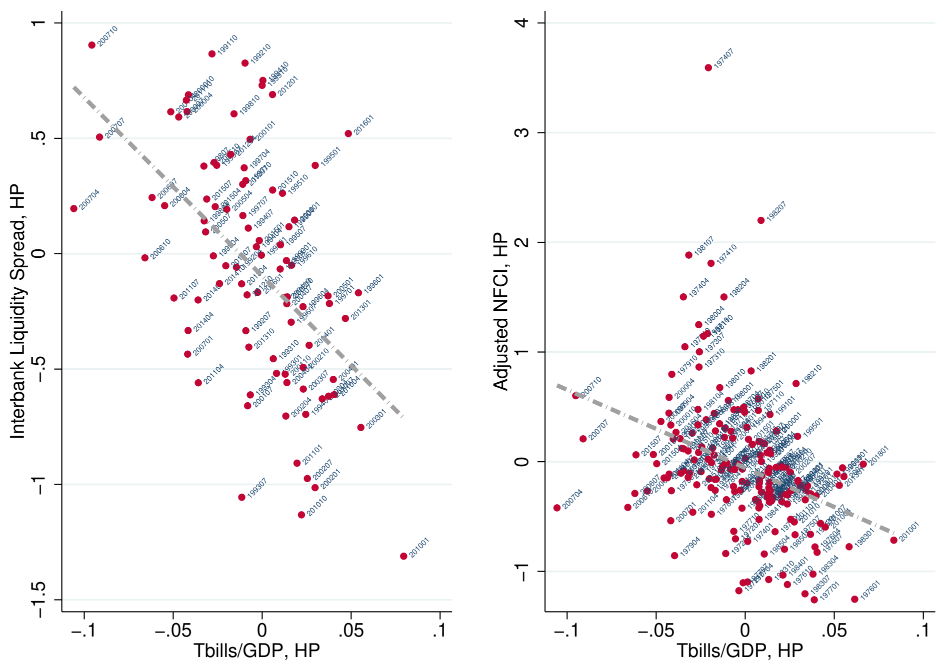

Figure 1: Increases in Treasury Bills Negatively Correlate with Tightness of Financial Conditions. Notes: The data on the interbank liquidity spread $(1991 : Q3 - 2016 : Q1)$ and the adjusted National Financial Condition Index $(1971 : Q1 - 2019 : Q1)$ are taken from Fred (data item: IBLSD678FRBCLE and ANFCI). The liability level of Treasury Bills for Federal government is used to calculate the Treasury bills-to-GDP ratio. All data series are HP-filtered. The outliers in 2008 and 2009 are removed.

[1] Adrian and Shin (2010) shows that repo contracts and financial commercial papers as a proportion of M2 rise from 25% in the early 1990s to 80% around 2007.

Figure 1 plots the interbank liquidity spread and the Adjusted National Financial Condition Index against the size of Treasury bills in the US economy. The interbank liquidity spread is defined as the difference between the 3-Month LIBOR and the 3-Month US Treasury Yield. The Adjusted National Financial Condition Index was developed to measure the tightness of financial conditions when compared with current economic conditions. Both of the variables are negatively correlated with Treasury-bills-to-GDP ratio. This result suggests that the supply of government debt may not be neutral in terms of financial conditions, especially for the interbank liquidity. More specifically, a higher supply of Treasury bills tends to improve the liquidity condition in the interbank market and the overall financial condition in the economy.

A direct implication of Figure 1 is that Ricardian Equivalence fails and that the way to finance government spending is important for the effectiveness of a fiscal stimulus. Following Nakamura and Steinsson (2014), I use historical data on military procurement to estimate the effects of government spending on state-level outputs. I divide sampling years into two groups depending on the growth rates of Treasury securities relative to historical median values. I am interested in whether the size of fiscal multipliers depends on the growth rates of Treasury securities. Column (1) in Table 1 shows the impact of Treasury bills. It implies that the fiscal multiplier is higher by 2.43 in the years with a higher growth rate of Treasury bills. Column (2) instead considers the impact of all government bonds. The result indicates that the fiscal multiplier is higher by 1.64 when the growth rate of all government bonds is high. Both of the estimates in columns (1) and (2) are significant and sizable, with the magnitude being larger in column (1).[2] The results suggest that the fiscal multiplier tends to be larger if government spending is accompanied by an increase in Treasury securities.

	Output	
	1. Treasury Bills	2. Total Gov Bonds
Military Spending	-0.36	0.73^{***}
	(0.775)	(0.227)
$\mathbb{1}_{HighSecurityGrowth}$ × Military Spending	2.43^{***}	1.64^{***}
	(0.871)	(0.614)
Year Fixed Effect	YES	YES
State Fixed Effect	YES	YES
Obs	1989	1989
R^2	0.324	0.317

Table 1: Fiscal multipliers Are Larger if Accompanied by Increases in Treasury Securities.

Motivated by the facts discussed above, I build a real business cycle model with liquidity

[2]The average fiscal multiplier estimated by Nakamura and Steinsson (2014) is roughly 1.5 in their baseline results. The larger estimate in Column (1) seems to suggest that it is the short-term government bonds, rather than overall government debts, that play an important role in determining the size of fiscal multipliers. This is consistent with the fact that Treasury bills, rather than the long-term government bonds, are the major repo collateral.

shocks and financial frictions in the banking sector. A key assumption of the model is that the borrowing constraints in the wholesale banking market take public debt as collateral. This is meant to capture the salient feature of repo contracts in reality. I show that if there is a sufficient supply of public debt, banks can borrow as much as they want from the wholesale banking market when facing idiosyncratic liquidity shocks. Hence, the well-functioning wholesale market helps banks to share idiosyncratic liquidity risks and the banking sector acts as a representative bank. In this equilibrium, Ricardian equivalence holds and the level of public debt is irrelevant.

However, if there is a shortage of public debt, I show that in the steady state the failure of Ricardian equivalence emerges through two channels. Firstly, there may be less investment because aggregate credit supply is limited by the supply of collateral. Banks with investment opportunities can not get a desirable amount of funds, while banks without investment opportunities have idle funds on their balance sheet. This liquidity mismatch reduces the aggregate credit supply and hence the aggregate investment in capital. As a result, the capital return rate is inefficiently high and there exists a wedge between the capital return and the deposit rate. Secondly, the shortage of public debt may increase capital accumulation through a financial speculation channel. More specifically, due to market segmentation, equity prices on non-investing islands will be higher than the prices on investing islands. I show that the dispersion of equity prices is larger if there is a shortage of public debt. The larger dispersion of asset prices creates incentives for banks to hold more equities so as to resell them at higher prices. These financial speculation activities would raise credit supply and encourage capital accumulation in the economy. The two channels work in different directions for capital investment. I show that under a mild shortage of public debt, the collateral channel dominates the speculation channel. Hence, the net impact of the shortage of public debt is to reduce capital investment. Under this scenario, higher public debt helps banks to extract excess returns from financial intermediations. At the aggregate level, increasing the supply of public debt improves the liquidity conditions of the banking sector, increases aggregate credit supply, and crowds in capital investment. This suggests that deficit-financed government spending has a positive externality stemming from improvements in financial intermediations.

I calibrate the model to the U.S. economy and discuss the model's performance in response to various types of shocks. I find that with wholesale interbank market frictions, a negative shock to investment opportunity more easily triggers financial recessions than a model with only retail deposit market frictions. Intuitively, a lower investment opportunity means that there are only a few islands where firms are able to make investment. The concentration of investment opportunities effectively makes the interbank borrowing constraint more binding for banks on these islands. As a result, aggregate investment drops and output is reduced by a sizable magnitude. In contrast, when the economy is buffeted by negative capital quality shocks, the response of the model with wholesale interbank market frictions is quite moderate when compared with the model in Gertler and Kiyotaki (2010). The reason is that the aggregate investment is constrained mainly by the size of public debt in the economy, which in fact provides countercyclical liquidity for the economy.[3] When incorporating the deposit market frictions in the model, the response of output to negative capital quality shocks is still muted. This is because, with the baseline calibration, it is the banks without investment opportunities that have higher leverage and are constrained by deposit market frictions. When hit by negative capital quality shocks, their weakened balance sheet conditions do not directly transmit into aggregate investment.

Lastly, I show that with the interbank market borrowing constraint, a debt-financed government spending shock can generate a larger stimulus effect than a tax-financed government spending shock. Intuitively, a debt-financed government spending package injects public debt as collateral into the interbank market and improves financial intermediations. It follows that the degree of liquidity mismatch within the banking sector is reduced and the credit supply becomes higher. As a result, aggregate investment is crowded in and the cumulative fiscal multiplier on output is larger than one. The size of the stimulus effect is shown to be decreasing in the degree of tax financing, providing additional support for tax smoothing.

Literature This paper is mainly related to four strands of literature. In the following, I briefly summarize each of these strands and discuss the contributions of my paper.

The first strand of literature studies the liquidity service provided by public debt (Barro (1974), Woodford (1990), Holmström and Tirole (1998), Aiyagari and McGrattan (1998)). In this literature, public debt is valued as liquid assets by either households or production firms. A higher supply of public debt is shown to increase the flexibility of private sectors in responding to variations in income or productivity. In this sense, public debt share similar role of store of value or buffer stock as money or rational bubbles(Bewley et al. (1980), Miao, Wang, and Zhou (2015), Wen (2015), Brunnermeier and Sannikov (2016)). Within this literature, my paper shares a similar approach to that used by Angeletos, Collard, and Dellas (2016) in which public debt can serve as collateral and provide liquidity service. However, Angeletos, Collard, and Dellas (2016) focus on the optimal fiscal policy where the government is facing the trade-off between liquidity benefits (on the production side) of higher public debt and the associating distortionary tax. My paper on the other hand focuses on the financial frictions in the banking sector and how the fricions affect the size of fiscal multiplier.

My paper is also related to the rapidly growing literature on banking (Gertler and Kiyotaki, 2010; Gertler, Kiyotaki, and Queralto, 2012; Gertler, Kiyotaki, and Prestipino, 2016). More specifically, Gertler and Kiyotaki (2010) and Gertler, Kiyotaki, and Queralto (2012) study how distortions of financial intermediations can affect the transmissions and the propagations of shocks. Compared

[3]As pointed out by Werning (2015), a similar mechanism also arises in McKay, Nakamura, and Steinsson (2016) since the size of public debt is kept at constant.

to these two papers, my paper seeks to explicitly and separately analyzes the wholesale banking market with repo contracts, while Gertler and Kiyotaki (2010) and Gertler, Kiyotaki, and Queralto (2012) focus on modeling both the retail and wholesale banking market frictions in a single incentive constraint. With richer banking market frictions, I show five types of equilibrium can arise in my full-fledged model, depending on the tightness of frictions in each of the two banking markets.

The third strand of literature that is related to my paper is on the size of the fiscal multiplier. Woodford (2011), among many others, points out that the size of the fiscal multiplier depends crucially on the responses of the interest rate. At the zero lower bound, the inaction of nominal interest rates to inflation lowers the real interest rate and makes the fiscal multiplier much larger. My paper investigates how the increased supply of government debt, which is a result of fiscal stimulus, can influence the response of interest rate through financial intermediation, and hence the size of the fiscal multiplier. To fucus on the role of public debt, I abstract my model from the nominal rigidity and monetary policy, and instead focus on the impacts of real rigidity due to financial frictions in the banking markets. My paper is also related to the empirical estimates of the fiscal multiplier. The motivating facts of my paper borrow the methods in Nakamura and Steinsson (2014) to show that the fiscal multiplier tends to be larger if it is accompanied by a higher growth rate of public debt. The improved efficiency of financial intermediation, resulting from public debt issuing, constitutes another micro-foundation of economic slackness that is discussed in Nakamura and Steinsson (2014).

The fourth strand of literature is on the causes of economic recessions and the evaluation of policy responses. For example, Del Negro et al. (2017) show that a shock to the liquidity of private paper helps to explain the great recession in 2008. They also argue that the liquidity facilities, adopted by the Fed, managed to prevent another round of recession. While they focus on liquidity shocks to private paper and the evaluations of policy responses, I focus on the liquidity service provided by Treasury securities and the size of the fiscal multiplier. Caballero and Farhi (2013); Caballero, Farhi, and Gourinchas (2016, 2017), Caballero and Farhi (2017) and Caballero (2018) highlight the shortage of safe asset as an explanation of the secular decline of the risk-free rate over the past few decades. They also argue that an acute form of a liquidity trap with endogenous risk premia can arise if there is a drop in safe asset supply or an increase in safe asset demand. In my paper, public debt is taken as safe assets and is especially valuable in financial intermediations.

In Section 2, this paper builds a real business cycle model with both frictions in the retail banking market and the wholesale banking market. In Section 3, to give more intuitions, I consider a special case where only the friction in the wholesale banking market matters. In Section 4, I examine a general case where frictions in both markets take effect. Section 5 ends with some concluding remarks.

2 Model

Consider an economy with a representative household and a continuum of islands. Following Gertler and Kiyotaki (2010), I assume that there is a continuum of firms and banks of mass unity located on the islands. Banks provide commercial loans to nonfinancial firms on the same island. In each period, investment opportunity arrives randomly to a fraction π_t^i of islands. On a fraction $\pi_t^n = 1 - \pi_t^i$ of islands, there are no investment opportunities. Only firms on the investing islands can acquire capital. The arrival of investment opportunities is i.i.d. across time and islands.

The idiosyncratic investment opportunities create liquidity shocks to banks. The wholesale banking market arises as a risk-sharing mechanism, through which banks with investment opportunities can borrow money from the others. If there is no friction in the wholesale banking market, investing banks can borrow as much as they need and the idiosyncratic investment opportunity shocks are perfectly diversified away. However, to the extent that financial frictions impede interbank borrowing, the risk sharing of idiosyncratic investment opportunities is also limited. As a result, a wedge between capital return rate and deposit rate emerges, showing the imperfections in financial intermediations.

In the following, I first describe the household's problem and the production side of the economy. Then I will discuss in detail the setup of the banking sector. I will keep the nonfinancial firms and households simple so as to highlight the impacts of the banking sector.

2.1 Household

There is a representative household with a unit mass of members. Within the household, there are $1 - f$ workers and f bankers. Workers supply labor in the national market and return their wage earnings to the household. Each banker manages a financial intermediary and transfers their non-negative dividends to the household. I assume that there is perfect consumption insurance within the household.

In the following, I use uppercase letters to denote aggregate variables and use lowercase letters to denote variables at the individual level. Let $D_{g,t}$ denote government bonds, D_t the deposits. The household can either hold government bonds or deposit funds in banks. But they cannot directly provide loans to nonfinancial firms because this requires some superior intermediation technology that is only available to banks. Without financial frictions, government bonds and deposits are perfect substitutes for the household. However, when government bonds are valued also for their collateral role, there is a liquidity premium on the price of government bonds and the return is lower. As a result, the household will only hold bank deposits in equilibrium. Without loss of generality, I assume that all of the government bonds are always held by banks.

Let W_t denote the wage rate, T_t the lump-sum taxes, R_t the rate of return on bank deposits, and Π_t the net profits from banks and nonfiancial firms. The household chooses consumption C_t, labor supply L_t and deposits D_t to solve the following problem:

$$\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left[\ln\left(C_{t+i} - \gamma C_{t+i-1}\right) - \chi \frac{L_{t+i}^{1-\epsilon}}{1+\epsilon} \right]$$
$$s.t.\ C_t + D_t = W_t L_t + \Pi_t - T_t + R_{t-1} D_{t-1},$$

where χ measures the degree of habit formation.

The first-order conditions for labor supply and consumption/saving are then given by

$$\mathbb{E}_t u_{C_t} W_t = \chi L_t^{\epsilon}, \tag{1}$$

$$\mathbb{E}_t \Lambda_{t,t+1} R_t = 1, \tag{2}$$

with

$$u_{C_t} \equiv \left(C_t - \gamma C_{t-1}\right)^{-1} - \beta\gamma \left(C_{t+1} - \gamma C_t\right)^{-1},$$
$$\Lambda_{t,t+1} \equiv \beta \frac{u_{C_{t+1}}}{u_{C_t}}.$$

2.2 Nonfinancial firms

There are two kinds of nonfinancial firms: final goods producers and capital goods producer.

2.2.1 Final goods producers

Assume that final goods producers on each island produce output using an identical Cobb-Douglas production technology with capital and labor as inputs. Capital is not mobile but labor is allowed to freely migrate across islands. In the following, I will first solve the firms' static labor choice problem and then characterize the optimal investment decision.

Due to constant-return to scale production technology and perfect mobility of labor, the final goods producers behave as a representative firm. Given aggregate capigal K_t and the wage rate W_t at period t, the representative firm chooses labor L_t to solve the following problem:[4]

$$A_t K_t^{\alpha} L_t^{1-\alpha} - W_t L_t.$$

The first-order condition for labor choice implies that

$$(1-\alpha) A_t (\frac{K_t}{L_t})^{\alpha} = W_t. \tag{3}$$

[4]One can equivalently solve individual firm's problem, sum up output and input decisions across firms, then verify the validity of the representative firm argument.

Substituting back into the firm's objective funtion, I obtain the capital return rate for each unit of capital

$$
\begin{aligned}
Z_t &\equiv \frac{A_t K_t^{\alpha} L_t^{1-\alpha} - W_t L_t}{K_t} \\
&= \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} A_t^{\frac{1}{\alpha}} W_t^{-\frac{1-\alpha}{\alpha}}. \qquad (4)
\end{aligned}
$$

After the production of goods, firms will make their investment decisions. In each period, investment opportunities arrive randomly to a fraction of π_t^i islands. For firms on investing islands, they can acquire total amount of capital I_t at price Q_t^i. However, for firms on non-investing islands, they are not able to make capital investment. Therefore, the law of motion for aggregate capital K_t is in turn given by

$$
\begin{aligned}
K_{t+1} &= \psi_{t+1}\left[I_t + \pi_t^i(1-\delta)K_t\right] + \psi_{t+1}\pi_t^n(1-\delta)K_t \\
&= \psi_{t+1}\left[I_t + (1-\delta)K_t\right], \qquad (5)
\end{aligned}
$$

where δ is the capital depreciation rate and ψ_{t+1} is the aggregate capital quality shock.

For simplicity, I assume that conditional on obtaining funds from banks, there are no further financial frictions on the firm sides. Therefore, firms on investing islands effectively issue new equity to finance their investment I_t. And each unit of equity is a state-contingent claim to future returns of one unit of capital, which are given by

$$
\psi_{t+1}Z_{t+1},\ (1-\delta)\psi_{t+1}\psi_{t+2}Z_{t+2},\ (1-\delta)^2\psi_{t+1}\psi_{t+2}\psi_{t+3}Z_{t+3}, ...
$$

With perfect competition, firms will invest to the point where the equity price is equal to the price of capital goods Q_t^i. This optimality condition also determines the demand for capital goods. Since the firm's equity is owned by bankers, the demand for capital goods is ultimately determined by the asset pricing condition of bankers. I will give more details in the following sections.

2.2.2 Capital Goods Producer

Capital goods producers are assumed to operate in the national market. They use final goods to produce capital, which is then sold to final goods producers with investment opportunities. The resulting profits are redistributed to the household. More specifically, they solve the following problem:

$$
\max_{\{I_s\}} \mathbb{E}_t \sum_{s=t}^{\infty} \Lambda_{t,s}\left\{Q_s^i I_s - \left[1 + f\left(\frac{I_s}{I_{s-1}}\right)\right] I_s\right\}, \qquad (6)
$$

where $f(\frac{I_s}{I_{s-1}})I_s$ captures the capital adjustment costs with $f(1) = f'(1) = 0$ and $f''(\cdot) > 0$.

The first-order condition implies that

$$
Q_t^i = 1 + f\left(\frac{I_t}{I_{t-1}}\right) + \frac{I_t}{I_{t-1}} f'\left(\frac{I_t}{I_{t-1}}\right) - \mathbb{E}_t\Lambda_{t,t+1}\left(\frac{I_{t+1}}{I_t}\right)^2 f'\left(\frac{I_{t+1}}{I_t}\right). \qquad (7)
$$

2.3 Banks

There are banks on each island that provide commercial loans to final goods producers on the same island. To finance lending, banks can raise funds either through retail deposit markets or wholesale interbank markets. However, there may exist financial frictions in both of the two markets.

At the beginning of each period, aggregate shocks realize. Then banks make decisions on their borrowings from households D_t and the holdings on government bonds $D_{g,t}$. The idiosyncratic investment opportunity then arrives randomly on each island. For those on investing islands, banks finance both the new and the existing equity at price Q_t^i. On non-investing islands, there is no issuance of new equity and banks provide loans only to the existing projects at price Q_t^n. Hence, there is higher credit demand on investing islands, which creates liquidity needs for the banks on these islands. These investing banks can raise additional funds from other banks through the wholesale interbank market. Those non-investing banks also benefit from the interbank market, since lending idle money through the market provides additional short-term returns.

For an individual bank on island of type $h \in \{i, n\}$, denote the net worth n_t^h, the interbank borrowing b_t^h, the deposit issuance d_t, the holdings of government bonds $d_{g,t}$ and the total value of commercial loans $Q_t^h s_t^h$. Then banks have the following flow-of-funds constraint:

$$d_{g,t}/R_{g,t} + Q_t^h s_t^h = n_t^h + b_t^h + d_t. \tag{8}$$

Let R_t, $R_{b,t}$, and $R_{g,t}$ be the deposit rate, the interbank interest rate, and the government bond return rate, respectively. Then the law of motion of bank net worth is given by

$$n_t^h = \left[Z_t + (1-\delta)Q_t^h\right]\psi_t s_{t-1} + d_{g,t-1} - R_{b,t-1}b_{t-1} - R_{t-1}d_{t-1}. \tag{9}$$

Notice that the return rate of equity holding s_{t-1} depends on the aggregate capital quality shock ψ_t, the capital return rate Z_t and the equity reselling price Q_t^h. Due to market segmentation, it is possible that the equity price Q_t^h differs across the types of island .

My key assumption for the wholesale interbank market is that the interbank borrowing uses government bonds as collateral. More specifically, I assume that the interbank borrowing satisfies

$$b_t^h \leq \phi d_{g,t}, \tag{10}$$

where ϕ captures the pledgeability of government bonds. In practice, ϕ is quite closed to one and $1-\phi$ is called the hair-cut rate. If the borrowing constraint does not bind, government bonds and deposits are perfect substitutes. Both of the assets are safe and provide the same nominal returns. However, if the borrowing constraint is binding, the rate of return on government bonds is lower than the deposit rate. This is because banks hold government bonds not only for their returns but also for their collateral values. Since the rate of return is too low, the household is not willing to hold government bonds. It follows that all of the government bonds end up in the hands of bankers. Without loss of generality, I assume banker's holding of government bonds is always non-negative.

Apart from the financial friction in the interbank market, there may also exist frictions in the deposit market. Both frictions will limit the bank's ability to raise funds. A simple way to model the deposit market friction is to assume that bankers may transfer a fraction θ of divertable assets to his or her family after obtaining the funds. Gertler and Kiyotaki (2010) assume that the total amount of divertable assets is given by $Q_t^h s_t^h - \omega b_t^h$. If $\omega = 0$, it means all the funds can be divertable. If $\omega = 1$, it means that banks cannot divert assets financed by interbank borrowing. In this paper, the total assets held by a bank is given by $Q_t^h s_t^h + d_{gt}$, with d_{gt} being used as collateral. Noting that the investment of government bonds is made earlier than the investment in equity, the cash in the hands of bankers is actually only $Q_t^h s_t^h$. Hence, I assume that $Q_t^h s_t^h$ constitutes all the divertable funds in our model.

Creditors recognize the bank's incentive to divert funds and hence will restrict the amount of funds they lend. To ensure the bank does not divert funds, the following incentive constraint must hold for each bank type:

$$V_t\left(s_t^h, d_{g,t}, b_t^h, d_t\right) \geq \theta Q_t^h s_t^h. \tag{11}$$

To maintain the tractability of the model, I assume that at the beginning of each period (before the realizations of aggregate shocks and the investment opportunity), banks can trade their assets so that their expected return of commercial loans are equalized across islands. More specifically, if there are favorable shocks to banks on some islands, the net worth of the banks on these islands will be higher. It follows that the banks are willing to supply more loans, driving down the expected future loan return. By allowing for trading assets at the begining of the periods, some of these banks sell their existing loans to the other banks that remain on the island and move their net worth to islands with higher expected loan returns. These transactions ensure that the valuation of assets and liabilities are equalized ex ante across islands. As a result, I do not need to keep track of the distribution of bank net worth across islands.

Noting that banks are eventually owned by households, the value function of bankers at the end of period $t-1$ satisfies the following Bellman equation

$$\begin{aligned} &V_{t-1}\left(s_{t-1}, d_{g,t-1}, b_{t-1}, d_{t-1}\right) \\ &\quad = \mathbb{E}_{t-1}\Lambda_{t-1,t}\sum_{h=i,n}\pi^h\left\{(1-\sigma)n_t^h + \sigma\max_{d_t,d_{g,t}}\left[\max_{s_t^h,b_t^h} V_t\left(s_t^h, d_{g,t}, b_t^h, d_t\right)\right]\right\}, \end{aligned} \tag{12}$$

where σ is the survival rate of bankers in each period. Introducing banker's turnover into the model is a convenient way to avoid banker's saving to overcome financial frictions. As a result, bankers will only pay dividends when it exits the market. I also assume that the choices of deposits d_t and government bonds $d_{g,t}$ are made before the realizations of investment opportunities.

To solve the problem, I conjecture that

$$V_t\left(s_t^h, d_{g,t}, b_t^h, d_t\right) = \mathcal{V}_{st}s_t^h + \mathcal{V}_{gt}d_{g,t}/R_{g,t} - \mathcal{V}_{bt}b_t^h - \mathcal{V}_{dt}d_t, \tag{13}$$

where $\mathcal{V}_{st}$,$\mathcal{V}_{gt}$, $\mathcal{V}_{bt}$ and $\mathcal{V}_{dt}$ are time-varying parameters. Taking $d_{g,t}$ and d_t as given, I first solve banker's problem after the realization of investment opportunities:

$$\max_{s_t^h, b_t^h} V_t\left(s_t^h, d_{g,t}, b_t^h, d_t\right), \tag{14}$$

subject to (10) and (11). And then I can turn to the optimal choice of deposit b_t and government bond d_{gt}. Let $\lambda_{w,t}^h$ be the Lagrangian multiplier associating with the borrowing constraint and $\bar{\lambda}_{w,t} = \sum_{h=i,n} \pi^h \lambda_{w,t}^h$ be the average of this multiplier. Similarly, let $\lambda_{r,t}^h$ be the Lagrangian multiplier associating with (11) and $\bar{\lambda}_{r,t}^h = \sum_{h=i,n} \pi^h \lambda_{r,t}^h$ be its average. The subscripts w and r stand for the wholesale and the retail banking market, respectively. Appendix A shows how to solve the banker's optimization problem.

The optimal choice for s_t^h is given by

$$(1 + \lambda_{r,t}^h)(\frac{\mathcal{V}_{st}}{Q_t^h} - \mathcal{V}_{bt}) = \theta\lambda_{r,t}^h + \lambda_{w,t}^h. \tag{15}$$

It states that $\frac{\mathcal{V}_{st}}{Q_t^h} \geq \mathcal{V}_{bt}$ with strict inequality holds if either interbank market or deposit market constraint binds. Thus, due to financial frictions, bank may not be able to exploit all the possible profits.

The first-order condition for d_t is given by

$$(1 + \bar{\lambda}_{r,t})(\mathcal{V}_{bt} - \mathcal{V}_{dt}) + \bar{\lambda}_{w,t} = 0. \tag{16}$$

This implies that the expected marginal cost of interbank borrowing $(1 + \bar{\lambda}_{r,t})\mathcal{V}_{bt} + \bar{\lambda}_{w,t}$ is higher than the expected marginal cost of deposit $(1 + \bar{\lambda}_{r,t})\mathcal{V}_{dt}$ if the incentive constraint binds at some states .

Similarly, the first-order condition for $d_{g,t}$ is given by

$$(1 + \bar{\lambda}_{r,t})\mathcal{V}_{gt} + \phi\bar{\lambda}_{w,t}R_{g,t} = (1 + \bar{\lambda}_{r,t})\mathcal{V}_{bt} + \bar{\lambda}_{w,t}, \tag{17}$$

where the left-hand side captures the marginal benefits of holding government bonds while the right hand side is the marginal cost. When the borrowing constraint in the interbank market binds on average, the collateral value of government bonds $\phi\bar{\lambda}_{w,t}R_{g,t}$ shows up on the left-hand side. On the other hand, higher holding of government bonds means banks need to borrow more in the wholesale market when investment opportunity arrives. This tends to tighten the borrowing constraint, which is captured by $\bar{\lambda}_{w,t}$ on the right-hand side. Reorganizing yields

$$(1 + \bar{\lambda}_{r,t})\mathcal{V}_{gt} = (1 + \bar{\lambda}_{r,t})\mathcal{V}_{bt} + (1 - \phi R_{g,t})\bar{\lambda}_{w,t}. \tag{18}$$

Comparing (16) and (18) once again shows that government bonds and deposits are perfect substitutes, i.e.

$$\mathcal{V}_{d,t} = \mathcal{V}_{b,t} = \mathcal{V}_{g,t},$$

if the interbank borrowing constraint is not binding, i.e. $\bar{\lambda}_{w,t} = 0$.

Substituting the optimality conditions (15), (16) and (18) into (12) and matching coefficients, I obtain the asset pricing conditions for equity s_t^h, government bonds $d_{g,t}$, interbank borrowing b_t^h and deposits d_t:

$$\mathcal{V}_{st} = E_t \Lambda_{t,t+1} \sum_{h=i,n} \pi^h \Omega_{t+1}^h \left[Z_{t+1} + (1-\delta) Q_{t+1}^h \right] \psi_{t+1}, \tag{19}$$

$$\mathcal{V}_{gt} = E_t \Lambda_{t,t+1} \sum_{h=i,n} \pi^h \Omega_{t+1}^h R_{gt}, \tag{20}$$

$$\mathcal{V}_{bt} = E_t \Lambda_{t,t+1} \sum_{h=i,n} \pi^h \Omega_{t+1}^h R_{bt}, \tag{21}$$

$$\mathcal{V}_{dt} = E_t \Lambda_{t,t+1} \sum_{h=i,n} \pi^h \Omega_{t+1}^h R_t, \tag{22}$$

where

$$\Omega_t^h = \left\{ (1-\sigma) + \sigma \left[(1 + \lambda_{r,t}^h) \mathcal{V}_{bt} + \lambda_{w,t}^h \right] \right\}.$$

2.4 Aggregation and Equilibrium

Recalling (3), the capital-labor ratio is the same across islands. It follows that the aggregate output Y_t can be expressed as

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}. \tag{23}$$

The aggregate demand for labor is given by

$$W_t = (1-\alpha) \frac{Y_t}{L_t}. \tag{24}$$

The resource constraint is given by

$$Y_t = C_t + G_t + \left(1 + f\left(\frac{I_t}{I_{t-1}} \right) \right) I_t. \tag{25}$$

The law of motion for aggregate capital is given by (5):

$$K_{t+1} = \psi_{t+1} \left[I_t + (1-\delta) K_t \right].$$

With the idiosyncratic nature of investment opportunities, the total amount of equity depends on the types of islands:

$$S_t^i = I_t + (1-\delta) \pi_t^i K_t, \tag{26}$$

$$S_t^n = (1-\delta) \pi_t^n K_t. \tag{27}$$

The total supply of equity is the sum of equity across islands:

$$S_t = S_t^i + S_t^n. \tag{28}$$

Due to the investment opportunity, there is higher demand for credit on investing islands

As discussed in Section 2.3, in each period a fraction $1-\sigma$ of bankers will exit the market and become workers, with all his or her retained earnings transferred to the household. To keep the number of bankers constant in equilibrium, I assume a mass $(1-\sigma)f$ of workers will become bankers. The existence of turnovers between bankers and workers is a convenient assumption to avoid bank's accumulating enough wealth to overcome financial frictions. Since bankers need net worth to operate in the market, I assume that the new bankers will receive start-up funds proportional to the total risky asset value of exiting bankers from the household. More specifically, the net worth of entering bankers of type h is given by

$$N_{y,t}^h = \xi \pi_t^h \left[Z_t + (1-\delta) Q_t^h \right] \psi_t S_{t-1}. \tag{29}$$

The net worth of existing bankers equals earnings on assets net debt payments made in the previous period, multiplied by the fraction that survive until the current period, σ:

$$N_{o,t}^h = \sigma \pi_t^h \left\{ \left[Z_t + (1-\delta) Q_t^h \right] \psi_t S_{t-1} + D_{g,t-1} - R_{t-1} D_{t-1} \right\}. \tag{30}$$

Hence, the total net worth of type h bankers is given by

$$N_t^h = N_{o,t}^h + N_{y,t}^h. \tag{31}$$

The balance sheet of the banking sector then implies

$$N_t^i + N_t^n + D_t = D_{g,t}/R_{g,t} + Q_t^i S_t^i + Q_t^n S_t^n. \tag{32}$$

The credit supply on each type of islands is possibly affected by frictions in the retail and the wholesale banking markets. I first notice that by the flow-of-funds constraint, the interbank market borrowing of type h banks can be expressed as

$$B_t^h = \pi_t^h D_{g,t}/R_{g,t} + Q_t^h S_t^h - N_t^h - \pi^h D_t, \tag{33}$$

which satisfies the wholesale market borrowing constraint

$$B_t^h \leq \phi \pi_t^h D_{g,t}, \tag{34}$$

with equity holds iff $\lambda_{w,t}^h > 0$.

For the retail banking market constraint, by combining (8), (11) and (13), I can rewrite the equity constraint as

$$\left(\theta - \left(\frac{\mathcal{V}_{st}}{Q_t^h} - \mathcal{V}_{bt}\right) \right) Q_t^h s_t^h \leq \left(\mathcal{V}_{gt} - \mathcal{V}_{bt} \right) d_{g,t}/R_{g,t} + \mathcal{V}_{bt} n_t^h - (\mathcal{V}_{dt} - \mathcal{V}_{bt}) d_t.$$

Integrating the above equation shows that

$$\left[\theta - \left(\frac{\mathcal{V}_{st}}{Q_t^h} - \mathcal{V}_{bt}\right)\right] Q_t^h S_t^h \leq (\mathcal{V}_{bt} - \mathcal{V}_{dt})\, \pi^h D_t + (\mathcal{V}_{gt} - \mathcal{V}_{bt})\, \pi^h D_{g,t}/R_{g,t} + \mathcal{V}_{bt} N_t^h, \tag{35}$$

with equality holds iff $\lambda_{r,t}^h > 0$.

Suppose that G_t and D_{gt} exogenously chosen by government, the government budget constraint satisfies

$$D_{gt}/R_{g,t} - D_{gt-1} = G_t - T_t. \tag{36}$$

As a baseline, I assume that government levies lump-sum tax T_t such that the supply of public debt does not change overtime, i.e. $D_{gt} = D_g$. In section 4.3, I will consider some alternative forms of fiscal rules.

Given the exogenous process of $\{A_t, \psi_t, G_t, D_{gt}\}$, a competitive equilibrium consists of 27 aggregate variables, $\{$ Y_t, K_t, C_t, L_t, W_t, I_t, S_t^i, S_t^n, S_t, Q_t^i, Q_t^n, N_t^i, N_t^n, D_t, Z_t, R_t, R_{gt}, R_{bt}, $\lambda_{w,t}^i$, $\lambda_{w,t}^n$, $\lambda_{r,t}^i$, $\lambda_{r,t}^n$, $\mathcal{V}_{st}$, $\mathcal{V}_{gt}$, $\mathcal{V}_{bt}$, $\mathcal{V}_{dt}$, T_t $\}$, such that 27 equations (23), (5), (25), (1), (24), (7), (26), (27), (28), (15), (15), (31), (31), (32), (4), (2), (20), (21), (34), (34), (35), (35), (19), (18), (22), (16), and (36) hold, and b_t^h satisfies (33).

3 Steady State with Interbank Borrowing Constraint

The model presented in the previous section contains frictions in both of the retail deposit market and the wholesale interbank market. Before the analysis of the full-fledged model, I first focus on the steady state where there is only the financial friction in the wholesale market,[5] i.e. $\theta = -\infty$. Without frictions in the retail market, I obtain $\lambda_{r,t}^i = \lambda_{r,t}^n = 0$. In the following, I will take the public-debt-to-capital ratio D_g/K as exogenous, and study its impact on the economy.[6]

Using (29), (30), (31) and (32), the interbank borrowing constraint (10) can be rewritten as

$$\delta + \pi^i(1-\delta)(Q^n - 1)(\sigma + \xi - 1) \leq \phi \frac{\pi^i}{\pi^n} \frac{D_g}{K}.$$

Suppose that $\sigma + \xi - 1 < 0$ as in the baseline calibration in Gertler and Kiyotaki (2010). It follows that the constraint tends to bind if the size of government debt D_g/K is lower, the pledgeability of government bond ϕ is lower, or the investment opportunity is more concentrated on some islands such that $\frac{\pi^i}{\pi^n}$ is higher.

Due to the simplicity of the model, I am able to solve the steady state in close form, which is summarized as follows:

[5]For a model with only the friction in the retail market, see Gertler and Kiyotaki (2010).

[6]It can be shown that the public-debt-to-output ratio D_g/Y is monotonically increasing in D_g/K when the interbank market borrowing constraint is mildly binding. Hence, it is equivalent to analyze the model in terms of D_g/K.

Proposition 1 *Suppose θ is sufficiently small such that the retail market borrowing constraint (10) is not binding. The steady state of the model can be summarized by the following two equations:*

$$Q^n = \frac{1 + \frac{\pi^n}{\pi^i}\bar{\lambda}_w}{1 - \bar{\lambda}_w}, \tag{37}$$

$$\phi \frac{\pi^i}{\pi^n} \frac{D_g}{K} \geq \delta + \pi^i(1-\delta)(Q^n - 1)(\sigma + \xi - 1), \tag{38}$$

where $\bar{\lambda}_w > 0$ if and only if (38) binds.

In addition, if there is a shortage of public debt such that

$$\phi \frac{D_g}{K} < \frac{\pi^n}{\pi^i}\delta, \tag{39}$$

then the borrowing constraint (38) binds. And the Lagrangian multiplier $\bar{\lambda}_w$ is given by

$$\bar{\lambda}_w = \frac{\delta - \frac{\pi^i}{\pi^n}\phi\frac{D_g}{K}}{\delta - \frac{\pi^i}{\pi^n}\phi\frac{D_g}{K} + (1-\delta)(1-\sigma-\xi)}. \tag{40}$$

If $1 - \sigma - \xi > 0$, then $\bar{\lambda}_w$ is decreasing in D_g/K. Other endogenous variables can be expressed as functions of $\bar{\lambda}_w$, accordingly.

Proof. See Appendix D. ■

Proposition 1 suggests that if there is enough public debt in the economy such that the borrowing constraint (38) does not bind ($\bar{\lambda}_w = 0$), the model is reduced to a canonical RBC model with

$$Z + (1-\delta) = R = R_g = R_b = 1/\beta. \tag{41}$$

Therefore, (41) implies that there is no wedge between capital return and deposit rate and financial intermediation is efficient. With enough public debt, Ricardian equivalence holds and changes in public debt have no impact on the real economy.

However, if there is no enough government bonds available for the use of collateral, then the interbank market borrowing constraint (38) will be binding. More specifically, equation (39) suggests that the constraint tends to bind if the size of public debt D_g/K is lower, the pledgeability ϕ is lower, or the investment opportunity is higher.

Under the mild condition $1 - \sigma - \xi > 0$, $\bar{\lambda}_w$ is decreasing in D_g/K. Intuitively, a higher supply of public debt as collateral helps investing banks to overcome their interbank borrowing constraint. The condition $1 - \sigma - \xi > 0$ ensures that there is an external financing needs of the banking sector to hold risky assets, after taking account the turnover of bankers.

Combining (37) with (40) shows that with lower $\phi D_g/K$, the equity price on non-investing islands Q^n is higher. Intuitively, banks on non-investing islands can either invest in interbank lending or provide commercial loans to nonfinancial firms. The lack of government bonds will force banks to invest more in nonfinancial firms, which raises the equity prices. The interaction between equity prices and interbank market friction can also be seen in (37), which shows that Q^n is monotonically increasing in the Lagrangian multiplier $\bar{\lambda}_w$.

I am interested in the impact of government bonds on the real block. The results are summarized in the following proposition:

Proposition 2 *Suppose that the assumptions in Proposition 1 hold. Then in the steady state, the deposit rate R, the government bond return R_g, and the interbank borrowing rate R_b are given by*

$$R = \frac{1}{\beta}, \tag{42}$$

$$R_g = \frac{1}{\beta + \phi \bar{\lambda}_w} < R, \tag{43}$$

$$R_b = \frac{1}{\beta}\left(1 - \bar{\lambda}_w\right) < R, \tag{44}$$

with $\bar{\lambda}_w > 0$ given by (40). The gross return on capital investment is given by

$$Z + (1-\delta) = \frac{1}{\beta}\left[1 + \frac{\pi^i}{\pi^n}\bar{\lambda}_w\left(1 - \beta(1-\delta)\left(\frac{1-\sigma}{1-\bar{\lambda}_w} + \sigma\right)\right)\right]. \tag{45}$$

Suppose the shortage of public debt is not too severe, i.e. $\bar{\lambda}_w$ is sufficiently small, then $Z + 1 - \delta > R$ and $Z \equiv Z(\bar{\lambda}_w)$ is decreasing in D_g/K.

Proof. See Appendix E. ■

Proposition 2 implies that the shortage of public debt will have impacts on asset returns on capital and government bonds. Recall that $\bar{\lambda}_w$ is decreasing in D_g/K by (40). As the supply of public debt D_g/K becomes lower, the government bond return and interbank borrowing rate are lower. With public debt shortage, the interbank borrowing from investing banks are limited by the amount of colleteral they have. The lower effective demand for interbank borrowing leads to lower interbank borrowing rate R_b. Since public debt is valued not only for the holding returns but also for its collateral status, banks therefore are willing to accept a lower return on public debt.

The result in Proposition 2 also shows that a public debt shortage will make capital return $Z + 1 - \delta$ higher than the deposit rate $R = 1/\beta$. In contrast to (41), the positive wedge between capital return rate and the deposit rate suggests that financial intermediation is not efficient. Intuitively, the wholesale banking market fails to provide perfect risk sharing of liquidity shocks due to the shortage of government bonds. Government can help with the imperfect financial intermediation by supplying more public debt. With a higher D_g/K, banks are less limited by the interbank borrowing constraint and the aggregate credit supply will be higher. It follows that capital investment increases and the capital return $Z + 1 - \delta$ is lower.

The assumption of mild public debt shortage is critical for the last part of Proposition 2. In fact, equation (45) shows that capital return can be non-monotonic in $\bar{\lambda}_w$. The non-monotonicity of capital return reflects two counteracting forces of public debt shortage: the collateral channel and the speculation channel. Firstly, a shortage of public debt reduces the credit supply on investing islands. This is because the banks are not able to raise funds from the interbank market due to the lack of collateral. Therefore, the collateral channel reduces the credit supply and capital accumulation.

Secondly, a shortage of public debt leads to speculations of equity prices on the non-investing islands. The equity prices on non-investing islands will be higher as banks are forced to shift their investment portfolio from interbank lending to providing commercial loans. With equity holdings, banks are entitled not only to production profits but also the gains from reselling the assets at the higher prices on non-investing islands. As a result, the speculation channel tends to increase credit supply and capital accumulation.

The two channels discussed above work in different directions, with the relative strength of the two channel depending on the degree of public debt shortage. More specifically, if there is no public debt shortage, i.e. (39) is violated, then $\bar{\lambda}_w = 0$. On the other hand, if there is no public debt supply in the economy, then $D_g/K = 0$ leads to

$$\bar{\lambda}_w = \frac{\delta}{\delta + (1-\delta)(1-\sigma-\xi)} \equiv \bar{\lambda}_w^{**}.$$

Hence, the range of $\bar{\lambda}_w$ is $[0, \bar{\lambda}_w^{**}]$. Note that the capital retun is given by (45). Let $\bar{\lambda}_w^*$ be the solution of $\partial Z(\bar{\lambda}_w)/\partial\bar{\lambda}_w = 0$. Then we have the following lemma:

Lemma 1 *Suppose* $1-\sigma-\xi > 0$. *Given* $\bar{\lambda}_w \in [0, \bar{\lambda}_w^{**}]$, $Z(\bar{\lambda}_w)$ *is increasing in* $\bar{\lambda}_w \in \left[0, \min\{\bar{\lambda}_w^*, \bar{\lambda}_w^{**}\}\right]$. *In addition, if* $\bar{\lambda}_w^* < \bar{\lambda}_w^{**}$, *then* $Z(\bar{\lambda}_w)$ *is decreasing in* $\bar{\lambda}_w \in \left[\bar{\lambda}_w^*, \bar{\lambda}_w^{**}\right]$.

Proof. See Appendix E. ■

When there is only mild shortage of public debt, i.e. $\bar{\lambda}_w \in \left[0, \bar{\lambda}_w^*\right]$, the collateral channel dominates. This is the scenario highlighted in Proposition 2. It follows that decreases in public debt lead to lower credit supply and reduce capital investment. However, if the interbank market is severely interrupted by the lack of public debt, i.e. $\bar{\lambda}_w \in \left[\bar{\lambda}_w^*, \bar{\lambda}_w^{**}\right]$, the speculation channel will be the driving force. Reselling assets on non-investing islands becomes attractive. Banks are willing to hold more equity so as to speculate on the reselling market. This tends to increase capital investment.

In the following, I will focus on the case where there is only a mild shortage of public debt and the collateral channel dominates. One reason is that the severe shortage case corresponds to public debt to GDP ratio lower than 30.1% under Gertler and Kiyotaki (2010) 's parameterization. The other reason is that our motivating facts in Section 1 seems to suggest that increases in public debt lead to higher output. This is consistent with the situation of a mild shortage of public debt.

Consider a permanent increase in government spending financed by issuing public debt. The supply of public debt helps to alleviate the financial friction in the wholesale banking market, which in turn boosts investment. Therefore, the fiscal multiplier is larger than the traditional model. More formally, combining (3) and (4), the aggregate output can be rewritten as

$$Y = \alpha^{\frac{\alpha}{1-\alpha}} A^{\frac{1}{1-\alpha}} Z^{-\frac{\alpha}{1-\alpha}} L. \tag{46}$$

In a canonical RBC model with lump-sum tax, Ricardian equivalence holds. The capital return rate Z given by (41) is independent of government spending and the ways of public financing. The impact of government expenditure on output only works through the wealth effect of labor supply. More specifically, higher government spending will crowd out consumption, encourage labor supply and increase output (Woodford, 2011). On the other hand, if labor supply is exogenous, government spending has no influence on the output level in the steady state. However, this is not the case when the wholesale market friction plays a role. A deficit-financed permanent increase in government spending raises the supply of public debt. According to Proposition (2), the wholesale market friction is alleviated. The increased credit supply boosts capital accumulation, reduces the capital return and raises output:

Proposition 3 *Suppose that assumptions in Propositions (1) and (2) hold and labor supply is exogenous. Then a permanent increase in government spending financed by issuance of public debt raises output level by increasing credit supply and capital investment.*

Proof. See Appendix F. ■

4 General Case and Dynamic Response

In this section I study the model performance in response to various types of shocks. In particular, I am interested in how the stimulus effects of government spending are affected by financial frictions and the supply of public debt. And to explore the robustness of the results, I allow for both the interbank market borrowing constraint (10) and the retail deposit market constraint (11) to take effects. To study these issues, it is needed to calibrate the model parameters first.

4.1 Calibration

Most of the parameters in this section are conventional and share the same values as in Gertler and Kiyotaki (2010). Table 2 provides a summary of my calibration. In the following, I will discuss some less standard parameter choices.

First, there are two parameters related to wholesale market frictions, namely the size of public debt D_g/Y and the repo haircut rate $1-\phi$. The calibration of the size of public debt is subject to some challenges. More specifically, all the public debt in my model are hold by the banking sector. In reality, a large fraction of public debt is held by Fed and foreign investors . Secondly, most of the collateral used in the repo contracts are Treasury bills, namely short-term government bonds.

Due to the these potential caveats, the aggregate public debt to GDP ratio may not be a good calibration target. With the concerns in mind, I adjust D_g/K such that the resulting public debt to GDP ratio is 55% or 75%, with the former (later) number being sufficiently small (large) so that the borrowing constraint (10) is (not) binding.

Table 2: Calibrated Parameters at Quarterly Frequency

Parameters	Value	Notes
β	0.990	Discount rate
γ	0.500	Habit parameter
χ	5.584	Relative utility weight of labor
ϵ	0.100	Inverse Frisch elasticity of labor supply
π^i	0.250	Probability of new investment opportunites
ξ	0.002	Transfer to entering bankers
σ	0.972	Survival rate of bankers
α	0.330	Capital share
δ	0.025	Depreciation rate
G/Y	0.200	Government spending share in output
Ω_K	0.500	Capital adjustment cost
ϕ	1.000	Pledgeability of public debt
ρ_ψ	0.660	Persistence of capital quality shocks
ρ_π	0.800	Persistence of investment opportunity shocks
ρ_G	0.800	Persistence of goverenment spending shocks
σ_ψ	0.01	Size of capital quality shocks
σ_π	0.20	Size of investment opportunity shocks
σ_G	0.01	Size of goverenment spending shocks

For the repo hair-cut rates, Gorton and Metrick (2012) show that those of the bilateral repo contracts are closed to 0% before the 2008 crisis. Krishnamurthy, Nagel, and Orlov (2014), on the other hand, provide evidence that the tri-party repo-haircut rates collaterized by U.S. Treasury are around 2% through out their whole sample periods ($2006m7$ to $2010m7$). For simplicity, I set $\phi = 1$ such that the calibrated hair-cut rates are zero. The steady state results remain robust to alternative calibration choices.

Second, to consider the existence of deposit market frictions, I also use different values for θ, i.e. the size of divertable funds. As in Gertler and Kiyotaki (2010), with higher θ, the deposit market frictions are more likely to be binding. In the following subsection, I am able to derive the threshold θ^* that determines the critical value of deposit market frictions. I set $\theta = 1.2\theta^* = 0.124$ to study the scenario where deposit market frictions are binding. I also consider the other case where the deposit market does not face financial frictions. This corresponds to the case where $\theta = 0.8\theta^* = 0.083$.

Lastly, for simplicity, I also assume a quadratic form of capital adjustment cost. More specifically, capital adjustment cost is given by

$$f(\frac{I_t}{I_{t-1}}) = \frac{\Omega_K}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2,$$

with $\Omega_K = 0.5$, which corresponds to the small adjustment cost case in Christiano, Eichenbaum, and Evans (2005).

I consider three types of shocks to the model, namely capital quality shocks, investment opportunity shocks and government spending shocks. More specifically, I assume

$$\hat{\psi}_t = \rho_\psi \hat{\psi}_{t-1} + \sigma_\psi \epsilon_{\psi,t},$$

$$\hat{\pi}_t = \rho_\pi \hat{\pi}_{t-1} + \sigma_\pi \epsilon_{\pi,t},$$

$$\hat{G}_t = \rho_G \hat{G}_{t-1} + \sigma_G \epsilon_{G,t},$$

where variables with hats denote log-deviations from steady states. The parameterizations for $\{\rho_\psi, \rho_\pi, \rho_G, \sigma_\psi, \sigma_\pi, \sigma_G\}$ are also given by Table 2. It is worth mentioning that with $\sigma_\pi = 0.20$, one standard deviation negative realization of $\epsilon_{\pi,t}$ reduces investment opportunity from 0.25 to $0.25 * 0.8 = 0.20$. In the following, I will show that this relatively mild reudction of investment opportunity can lead to sizable recession if the economy features with public debt shortage.

4.2 Steady States with Both Types of Frictions

In this section, I will numerically show that there are five types of equilibrium in the economy, depending on the tightness of financial frictions in the retail and the wholesale banking markets. Figure 2 shows the division of equilibrium regions.

If there is no frictions in the both of the retail and the wholesale markets, the model is reduced to a canonical RBC model. In this case, Ricardian equivalence holds. This is also the case if the size of public debt D_g/K is large enough and the size of divertable funds θ is small enough. As a result, the existing frictions in the wholesale interbank and the retail deposit market do not affect the equilibrium. This type of equilibrium corresponds to region ① in Figure 2. I have the following proposition:

Proposition 4 *Suppose that there are borrowing constraints (10) and (11) in the wholesale and the retail banking markets, respectively. The borrowing constraints are not binding if and only if the size of government bonds and the incentive constraint satisfy the following conditions:*

$$D_g/K \geq \frac{1}{\phi}\frac{\pi^n}{\pi^i}\delta, \text{ and } \theta \leq \frac{\pi^i \xi}{[\delta + \pi^i(1-\delta)](\beta - \sigma)}. \tag{47}$$

Proof. See Appendix G. ■

If either of conditions in (47) of Proposition 4 is violated, the characterization of steady state will be more complicated. In particular, numerical solution is needed to decide which type of islands will have retail market constraint (11) being binding.

If government bonds are abundant such that D_g/K is large and the interbank borrowing constraint (10) is not binding, it is still possible that the incentive constraint (11) in the deposit market binds. This case corresponds to the one analyzed by Gertler and Kiyotaki (2010), i.e. region ② in Figure 2. It can be shown that the Lagrangians of the borrowing constraints satisfy $\lambda_r^i > \lambda_r^n \geq 0$. This indicates that the retail market constraint on the investing islands is always binding, while the one on the non-investing islands is relatively less binding. The intuition is that the new equity supply on the investing islands raises the leverage of investing banks and therefore tightens the retail market constraint.

Another simple situation is the case where only the wholesale market constraint binds. This is the case I study in Section 3, i.e. region ③ in Figure 2. In this case, the shortage of public debt prevent banks on the investing islands from supplying funds to nonfinancial firms. Without necessary funding, nonfinancial firms are forced to cut back investment. The capital returns are still higher than the deposit rates, signaling failures of financial intermediations. In this case, public debt are expecially valuable because it enables its owner to channel funds from households to nonfinancial firms, earning the excess returns between capital returns and the deposit rates.

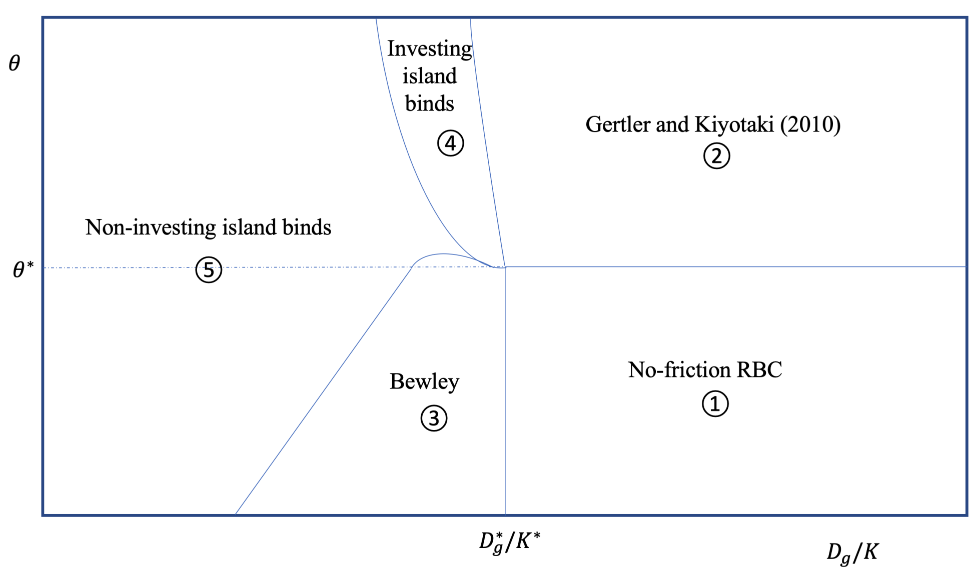

Figure 2: Equilibrium with Different Tightness of Financial Frictions

A more complicated case is where both of the retail and the wholesale banking constraints, i.e. (10) and (11), are binding. More specifically, it is hard to give an analytical solution to pin down the dependence of the retail market constraint (11) on the types of islands. Turing to numerical solutions, I find that if the size of government bonds is small but closed to the threshold D_g^*/K^*, only the banks on investing islands will be binding by the constraint in the retail market, corresponding to region ④. Intuitively, the bank leverage on investing islands is higher because there are more equity in equilibrium. This makes the divertable assets larger and tightens the retail deposit market constraint. However, I also find that Blanchard-Kahn condition is violiated (Blanchard and Kahn, 1980) in this type of equilibrium.[7]

The equilibrium property changes if the shortage of government bonds is more pronounce, as in region ⑤. In this case, it is on the non-investing islands that the retail market constraint (11) binds. The reason is that the shortage of public debt have a strong impact on the portfolio choice of non-investing banks. More specifically, due to the lack of public debt as collateral, the non-investing banks have to cut back their interbank lending and buy more existing equity. This pushes up the equity price on the non-investing islands and hence the bank leverage. As a result, the retail market constraint binds on non-investing islands. For the purpose of robustness check, I will focus on this equilibrium instead of region ④.

4.3 Dynamic Response to Shocks

Investment Opportunity Shocks I first consider a negative shock to investment opportunity. This experiment is meant to capture the observation that in recessions both banks and firms have less investment opportunity. Alternatively, the investment shocks can be interpreted as a reduced-form uncertainty shock(e.g. Bloom et al., 2018), without directly modelling the underlying shock process.

Figure 3 presents the impulse reponses to the investment opportunity shock. After the realization of the negative shock, the probability of having investment opportunity $\hat{\pi}_t^i$ is reduced from $\pi_t = 0.25$ to $\pi_t = 0.20$ in the first quarter, and then gradually converges to steady state within roughtly 25 quarters. Consider the model with only the interbank borrowing constraint being binding. The result is captured by the red dash-dotted line in Figure 3. With a shortage of public debt and the resulting imperfect interbank market, I find that the negative investment opportunity triggers a sizable decline (roughly 3%) in output. The output decline is driven by the falls in investment. In contrast, if there is no any frictions (i.e. no-friction RBC model) or there is only the retail market friction (i.e. the model in Gertler and Kiyotaki (2010)), output response is muted or much more milder. For these two cases, the model response is captured by the black solid line and the black dotted line, respectively.

As discussed, with public debt shortage, a decline in investment opportunity generates a much stronger recession. To see the intuition, it should be noted that a lower investment opportunity

[7] With the parameter values in Table 2, I adjust the combination of $\{D_g/K, \theta\}$ such that the steady state corresponds to region ④. For a quite wide range of $\{D_g/K, \theta\}$, none of the combinations satisfy the Blanchard-Kahn condition.

π_t means that the aggregate investment is concentrated on only a few islands. On these investing islands, the credit demand is higher but the public debt held by investing banks is lower. In the no-friction RBC model or the model in Gertler and Kiyotaki (2010), this does not matter much since the well-functioning wholesale interbank market helps to reallocate the funds among banks. However, if there is a shortage of public debt, the interbank reallocation of funds are limited by the amount of collateral. As a result, the wholesale interbank market frictions are tightened, which show up as upward jumps in $\lambda^i_{w,t}$. It follows that the credit supply falls and the aggregate investment drops by roughly 15%. Consumption arises initially, due to lower real interest rates. After a few quarters, consumption becomes negative due to the dominating force of the income effect.

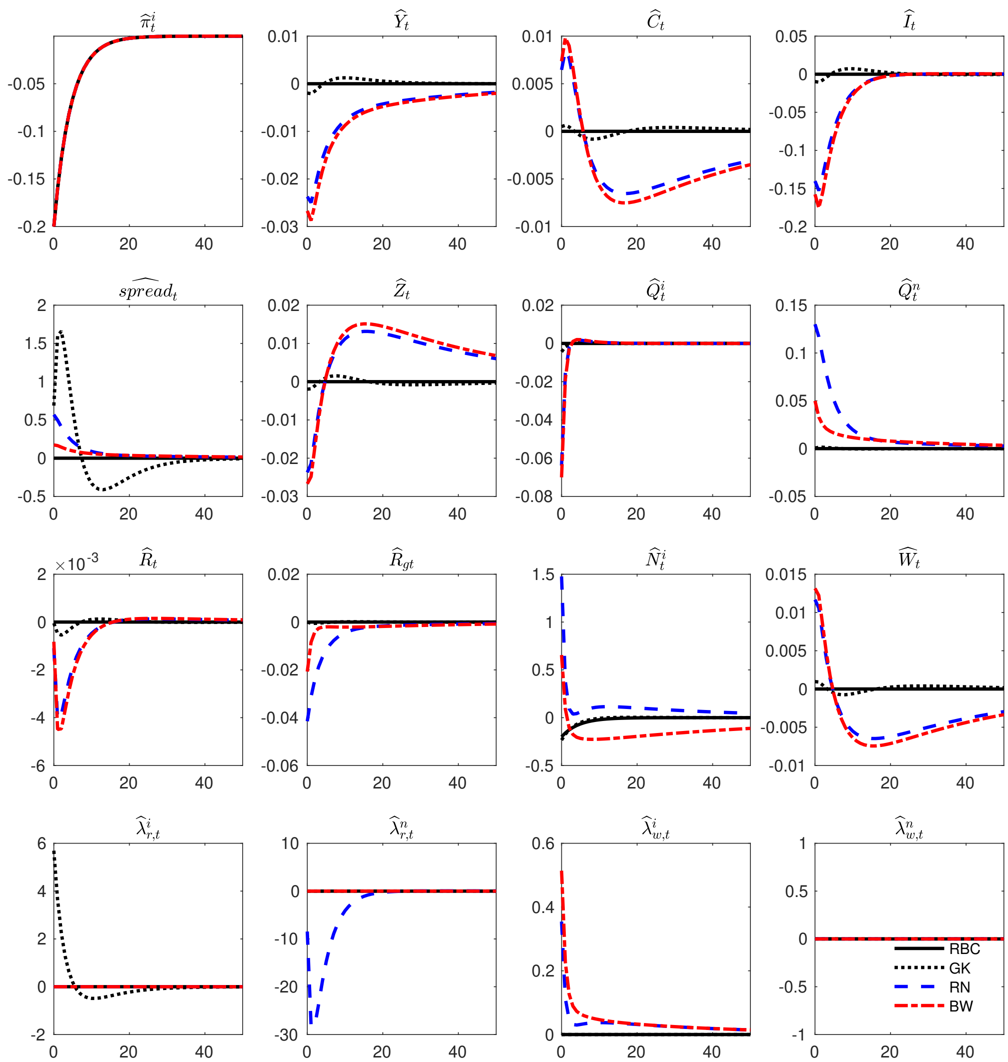

Figure 3: Dynamic Effects of a Negative Investment Opportunity Shock. Notes: The red dash-dotted line represents the response of the model with only the interbank borrowing constraint; the blue dash line shows the response of the model with frictions in both the interbank and the retail markets; the black dotted line represents the model with only the the retail market constraint; the black solid line represents the no-friction RBC model.

For the impacts on financial intermediation, Figure 3 also plots the response of return spread between capital investment and deposit. In the no-friction RBC model captured by the black solid line, the idiosyncratic investment opportunity shocks are perfectly diversified away. As a result, various asset returns are always equalized and the return spread remains at zero. However, with wholesale interbank market frictions, the positive return spread arises due to banks' failure in channeling funds to nonfinancial firms. It is also interesting to notice that the equity price on investing island falls while the equity price on non-investing islands arises. Therefore, a negative investment opportunity generates higher dispersion of equity prices. Intuitively, the equity price Q_t^i on investing islands falls because the reduction of investment opportunity makes the supply of capital goods relatively higher than the demand. On the other hand, the lack of public debt as collateral forces the banks on non-investing islands to shift from interbank lending to investing in existing equity, which drives up Q_t^n. Therefore, a shortage of public debt in this case leads financial speculation since the higher equity price does not channel funds directly into investment.

It is worth noting that the responses of output, consumption and investment are robust to the incorporation of retail deposit market friction. The response of the model with both frictions in the retail market and the wholesale market is captured by blue dash line in Figure 3. It can be seen that the impulse response is quantitatively similar to the one with only the interbank market constraint. However, the response of asset prices can be different. More specifically, the equity price of non-financial firms $\hat{Q}_t^n$ is higher and more responsive. This is due to the interaction of the frictions in both markets. More specifically, the negative investment opportunity shock redistributes the wealth from investing banks to non-investing banks. But due to the lack of collateral, the non-investing banks are not willing to lend out their money in the interbank market. Instead, they invest on the existing equity of non-financial firms on the non-investing islands, which pushes up the price Q_t^n. This increases the net worth of banks on the non-investing islands N_t^n. Recall that these banks are subject to retail deposit market friction. The increase in N_t^n helps to alleviate the constraint and increase their credit supply, which further pushes up Q_t^n. As a result, the retail market constraint amplifies the impact on the equity price Q_t^n.

Capital Quality Shocks Compared with investment opportunity shock, capital quality shock is a more conventional modeling device to trigger financial recessions. The capital quality shock affects the economy not only through its direct impact on the quality-adjusted capital stock but also through the deterioration of agent's balance sheet. The impaired balance sheet can further affect financial intermediations and reduce output, introducing an amplification mechanism into the model (Gertler and Kiyotaki, 2010).

Consider a negative shock to capital quality, which introduces a -1% decline in capital quality. The impulse responses are presented in Figure 4. Suppose there is only borrowing constraint in the retail deposit market, as discussed by Gertler and Kiyotaki (2010), the initial decline in capital quality is able to generate a sizable reduction in output. The impulse response is captured by the black dotted line in Figure 4. The intuition is that the negative capital quality shock reduces the net worth of banks, restricting the funding from the deposit market that is available to the banks. Because the leverage is higher for banks on investing islands, the borrowing constraint is more restrictive for these banks. This is the reason why the Lagrangian multiplier for the retail deposit market constraint on investing islands increases when the economy is hit by the negative capital quality shock.

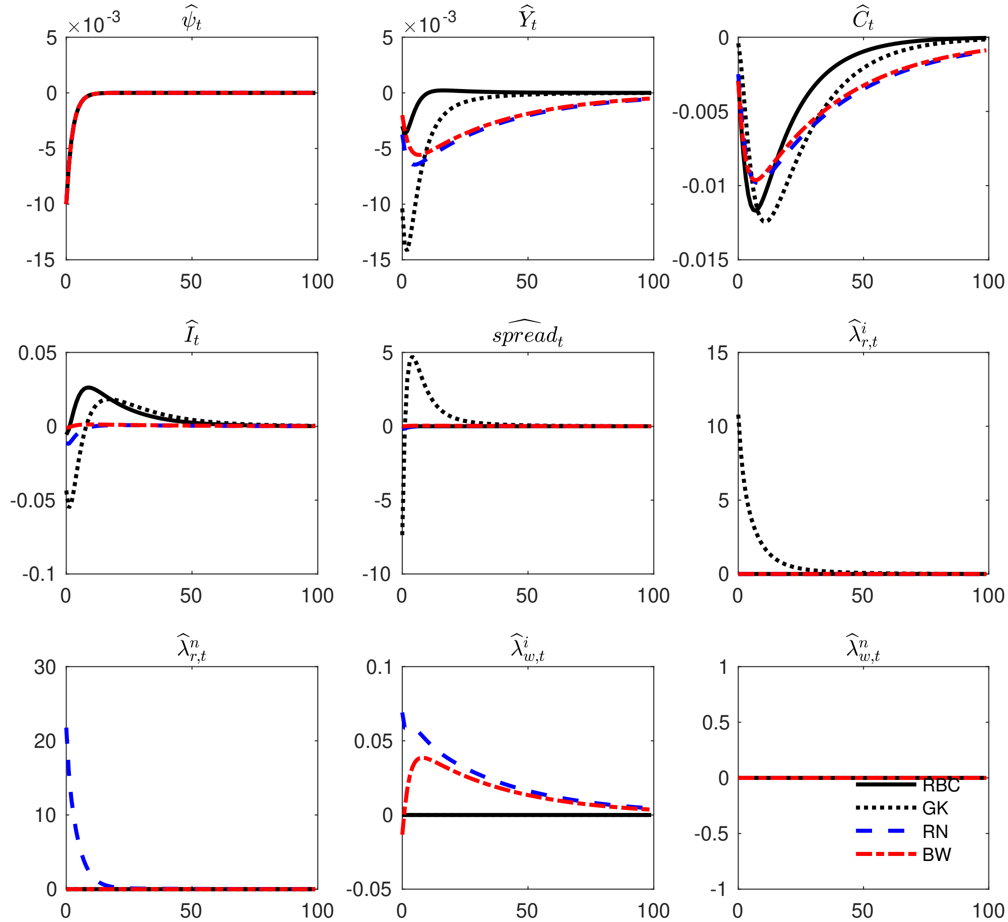

Figure 4: Dynamic Effects of a Negative Capital Quality Shock. Notes: The red dash-dotted line represents the response of the model with only the interbank borrowing constraint; the blue dash line shows the response of the model with frictions in both the interbank and the retail markets; the black dotted line represents the model with only the the retail market constraint; the black solid line represents the no-friction RBC model.

Now consider the model with only the wholesale market constraint (10) being binding. The impulse response is shown by the red dash-dotted line in Figure 4. Investment, which is supposed to increase as in the RBC model (black solid line), is now limited by the amount of public debt in the economy. More specifically, $\lambda^i_{w,t}$, the Lagrangian multiplier associated with the wholesale interbank borrowing constraint, rises after the negative capital quality shock realizes. This suggests that banks on the investing islands are not able to provide enough funding to nonfinancial firms. The decline in credit supply then reduces aggregate investment. Without amplification effects through the bank balance sheet channel highlighted by Gertler and Kiyotaki (2010), there is only a mild decline in output and investment.

Interestingly, when there are two types of financial constraint, i.e. (10) and (11), the negative impacts of the capital quality shock on output is smaller than the case with only one financial friction, i.e. the retail market constraint. The is essentially because the wholesale interbank market friction, once incorporated in the model, becomes the dominating force that determines the banks' investment behavior. More specifically, the parameterization of the model correspond to region ⑤ in Figure 2. This indicates that the retail deposit market constraint is binding on non-investing islands but not for banks on the investing islands. It follows that the only binding constraint for investing banks is the lack of public debt as collateral to borrow in the interbank market. Hence, the negative capital quality shock does not affect the ability of investing banks to borrow in the deposit market. Instead, the credit supply of investing banks are constrained by the interbank borrowing, which in turn is due to the shortage of public debt. Therefore, the real activities in the model are expected to behave similar to the ones with only wholesale interbank market friction. In contrast, for banks on non-investing islands, the negative capital quality shock reduces their net worth and their demand for equity. This only serves to reduce the equity price Q^n_t, without direct impacts on aggregate investment since firms on non-investing islands do not make capital investment.

The numerical exercise in Figure 4 shows that capital quality shock may not be able to generate sizable recession in a model with wholesale interbank market borrowing constraint. When banks are constrained by interbank borrowing, the shortage of public debt becomes the dominating force that pins down aggregate investment. The capital quality shock does affect the balance sheet of non-investing banks. But it is less relevant for banks with investment opportunity, since their ability to raise funds is not limited by their balance sheet but rather by the lack of public debt as collateral.

Government Spending Shocks Since the 2008 Great Recession, there is a burgeoning literature on fiscal multiplier pointing out that the presence of price stickiness, hand-to-mouth workers, zero lower bound for the nominal interest rate, among many other factors, can generate larger fiscal multipliers. In constrast, this paper shows that fiscal multiplier can be larger if it is financed by raising public debt. By issuing public debt, the government is also effectively supplying more collateral into the interbank market. Therefore, government spending has an additional positive externality stemming from the improvements of financial intermediations.

Figure 5 presents the impulse responses to a 1% government spending shock. The black solid line captures the impulse response of the no-friction RBC model. In a non-friction RBC model, increase in government spending crowds out investment and consumption, leading to a positive fiscal multiplier but with the size being smaller than 1. Since Ricardian equivalence holds, the type of financing for government spending does not matter.

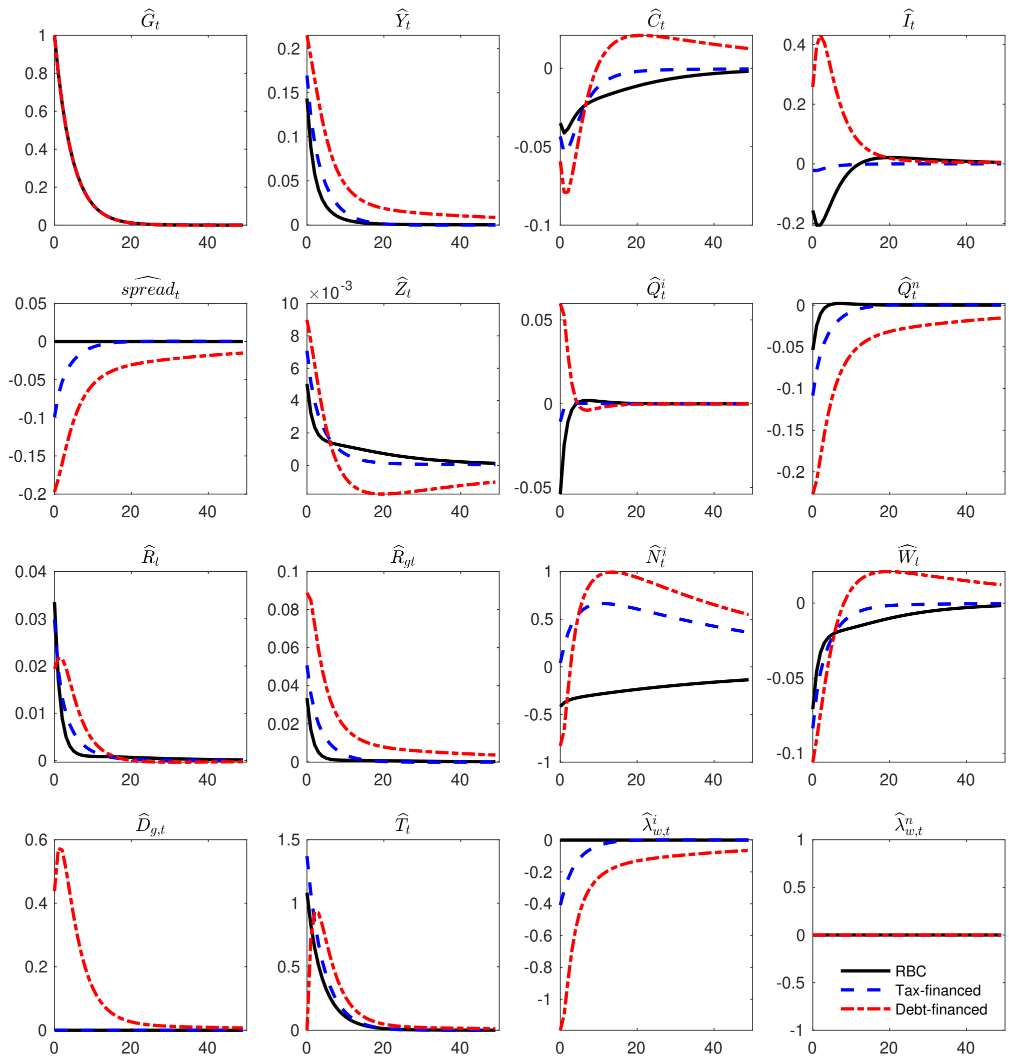

Figure 5: Dynamic Effects of a Positive Government Spending Shock. Notes: The blue dash line and the red dash-dotted line represent the impulse responses of the model with only the interbank borrowing constraint, with government spending financed by lump-sum tax or partially by public debt; the black solid line represents the impulse response of the no-friction RBC model.

Now consider the model with the interbank borrowing constraint (10). In this model, Ricardian equivalence fails due to the existence of financial frictions. It follows that the type of financing matters for the stimulus effect of government spending. Suppose that government spending is financed by lump-sum tax such that the level of public debt is kept constant, i.e. $D_{g,t} = D_g$. The impulse response is capture by the blue dash line in Figure 5. The stimulus effect of government spending is slightly larger than the one in the no-friction RBC model, since the crowding out effect on consumption and investment is smaller. Intuitively, with lower aggregate investment, the resulting lower aggregate credit demand effectively makes the interbank borrowing constraint less binding. Hence, the credit supply is relative higher and serves to moderate the decline in investment and investment.

Suppose that the government spending is partially financed by the lump-sum tax. Consider a fiscal rule as follows:

$$T_t - T = \phi_d \left(D_{g,t} - D_g \right), \tag{48}$$

where ϕ_d captures the responsiveness of lump-sum tax to the oustanding real debt. For our baseline calibration, set $\phi_d = 0.5$ such that 47.3% of the deviation of public debt from its steady state is corrected via lump-sum tax in each quarter. The red dash-dotted line in Figure 5 shows the impulse response. Compared with the tax-financed government spending, the debt-financed government spending creates a larger and more persistent stimulus effect on output. This is because the increase in public debt helps to alleviate the liquidity mismatch in the interbank market. The reallocation of funds from non-investing banks to investing banks raises the equity price Q_t^i on investing islands, while it also reduces the equity price Q_t^n on non-investing islands. The spread between capital investment return and deposit rate drops, and the aggregate investment is crowded in. Although there is slightly larger crowding out effect on consumption initially, consumption rises above the steady state after the initial declines.

The impulse responses in Figure 5 suggest that a debt-financed government spending has strong stimulus effect on investment, consumption and output. To see the robustness of this finding, I adjust the model by varying the tax responsiveness parameter ϕ_d and by incorporating the deposit market borrowing constraint (10). The results are presented in Figure 6, with the red solid line showing how the fiscal multipliers vary with the tax responsiveness parameter ϕ_d. The stimulus effect of government spending on output is robustly larger than 1 but declining in ϕ_d. Intuitively, the increase in public debt will converge to its steady state at a faster rate if ϕ_d is larger. It follows that the positive externality on financial intermediation will thus quickly disappear, leading to a smaller stimulus effect. Incorporating the deposit market borrowing constraint does not change the results, which is shown by the blue dash line in Figure 5. This is because the retail market borrowing constraint is not binding for investing banks, and their investment behavior is still mainly constrained by the lack of public debt as collateral.

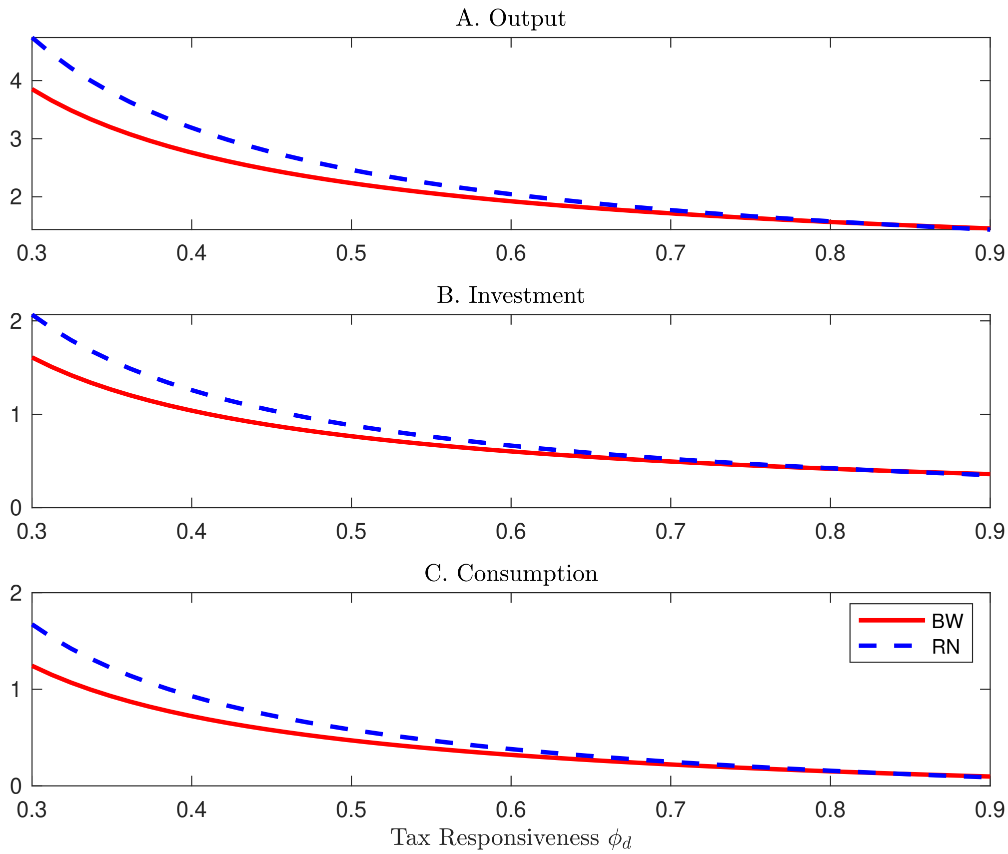

Figure 6: Cumulative Fiscal Multipliers as ϕ_d Varies. Notes: The cumulative fiscal multiplier for output is defined as the ratio of total output response over the total government spending. The cumulative fiscal multipliers for consumption and investment are defined similarly. The red solid line presents the multipliers for the model with only interbank borrowing constraint; the blue dash line presents the multipliers for the model with the borrowing constraint in both of the wholesale interbank market and the retail deposit market.

5 Conclusion

Studying the non-neutrality of public debt and the stimulative effects of fiscal policy is especially important since the Great Recession. In this paper, I provide suggestive evidence that the supply of public debt is positively correlated with financial conditions. I also find that the fiscal multiplier tends to be larger if government spending is accompanied by higher growth of Treasury securities in the economy. Given these motivating facts, I build a real business cycle model with financial frictions in the banking sector. The key assumption of the model is that banks have to pledge public debt as collateral to raise funds in the market. As a result, changes in public debt have real impacts on the economy by affecting financial intermediations.

For the steady state of the model, I show that if the supply of public debt is lower than a certain threshold, Ricardian equivalence fails. The shortage of public debt has impacts on the economy through two channels: the collateral channel and the financial speculation channel. Firstly, the lack of public debt as collateral generates liquidity mismatch in the banking sector. As a result, there is insufficient credit supply and capital is underaccumulated. Secondly, financial speculations arise because non-investing banks are forced to shift their investment portfolio from interbank lending to purchasing existing equity. As a result, asset prices on non-investing islands are higher, which raises net worth of banks and hence leads to higher aggregate credit supply. For capital accumulation, the two channels work towards different directions. I show that with mild shortage of public debt, the collateral channel is the dominating force. In this case, a permanent increase of debt-financed government spending can raise aggregate output even if labor supply is exogenous. This is because the increase in public debt helps to improve financial intermediation.

With the model calibrated to the U.S. economy, I study the model performance in response to various types of shocks. I find that with wholesale interbank market frictions, a negative shock to investment opportunity is able to trigger sizable declines in economic activities; on the other hand, a negative capital quality shock is less able to do so. This finding is robust even if the banks also suffer from borrowing constraints in the retail deposit market. Lastly, I show that with the interbank market borrowing constraint, a debt-financed government spending shock is able to generate a larger stimulus effect than a tax-financed government spending shock. This is because investment and consumption are crowded in due to the improvements in financial intermediations. In addition, I find that the size of the cumulative fiscal multiplier is decreasing in the magnitude of tax adjustment.

For tractability, the model in this paper is higly stylized and I have abstracted away from other source of safe asset supply and the endogenous determination of public debt as safe asset. Considering close substitutes of public debt, as well as their endogenous supply, may change the quantitative performance of the model, but may not qualitatively change the key message of this paper. In addition, I do not consider the presence of some New Keynesian elements, such as price stickiness, hand-to-mouth workers, and the zero lower bound on nominal interest rate. Studying how shortage of public debt interacts with these elements is quite interesting for future research.

One may argue that the public debt to GDP ratio has rose to over 100% since 2008, and it is unnecessary to worry about the shortage of public debt. However, roughly half of U.S. public debt is held by foreign and international investors.[8] As for the banking sector, the type of Treasury secuties used as collateral in repo contracts are mostly short-term goverment bonds. Moreover, public debt as typical "safe" assets may be subject to unexpected sentiment shocks(He, Krishnamurthy, and Milbradt, 2019). In fact, the Euro sovereign debt crisis and the liquidity dry-ups of U.S. mortgage-backed securities around 2008 also shows that the assets perceived to be safe may turn out to be risky and lose their collateral values. Given these concerns, the potential impacts of public debt shortage deserves further careful research.

[8] More specifically, the foreign holding share of U.S. Treasury securities is estimated to be 48% in 2014, 48% in 2015. Recently, the share is declined to be 39% in 2018 but it is quite sizable. See `https://fas.org/sgp/crs/misc/RS22331.pdf` for more details.

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For Online Publication Appendix

A Banker's Problem

Suppose that $V_{t-1}(s_{t-1}, d_{g,t-1}, b_{t-1}, d_{t-1})$ is banker's value function at the end of period $t-1$. Then banker's problem can be summarized as

$$\begin{aligned} & V_{t-1}(s_{t-1}, d_{g,t-1}, b_{t-1}, d_{t-1}) \\ & = \mathbb{E}_{t-1}\Lambda_{t-1,t}\textstyle\sum_{h=i,n}\pi^h \left\{(1-\sigma)n_t^h + \sigma \max_{d_t,d_{g,t}}\left[\max_{s_t^h,b_t^h} V_t\left(s_t^h, d_{g,t}, b_t^h, d_t\right)\right]\right\}, \end{aligned}$$

$$s.t. \quad V_t\left(s_t^h, d_{g,t}, b_t^h, d_t\right) \geq \theta Q_t^h s_t^h, \tag{A.1}$$

$$b_t^h \leq \phi d_{g,t}. \tag{A.2}$$

$$d_{g,t}/R_{g,t} + Q_t^h s_t^h = n_t^h + b_t^h + d_t, \tag{A.3}$$

$$n_t^h = \left[Z_t + (1-\delta)Q_t^h\right]\psi_t s_{t-1} + d_{g,t-1} - R_{b,t-1}b_{t-1} - R_{t-1}d_{t-1}. \tag{A.4}$$

Conjecture that the solution of the value function is given by

$$V_t\left(s_t^h, d_{g,t}, b_t^h, d_t\right) = \mathcal{V}_{st}s_t^h + \mathcal{V}_{gt}d_{g,t}/R_{g,t} - \mathcal{V}_{bt}b_t^h - \mathcal{V}_{dt}d_t, \tag{A.5}$$

where $\mathcal{V}_{st}$,$\mathcal{V}_{gt}$, $\mathcal{V}_{bt}$ and $\mathcal{V}_{dt}$ are time-varying parameters.

Then the Lagrangian is given by

$$\mathbb{L} = V_t\left(s_t^h, d_{g,t}, b_t^h, d_t\right) + \lambda_{r,t}^h\left[V_t\left(s_t^h, d_{g,t}, b_t^h, d_t\right) - \theta Q_t^h s_t^h\right] + \lambda_{w,t}^h(\phi d_{g,t} - b_t^h).$$

Substituting (A.5) into the Lagrangian and reorganizing yield

$$\begin{aligned} \mathbb{L} =& V_t\left(s_t^h, d_{g,t}, b_t^h, d_t\right) + \lambda_{r,t}^h\left[V_t\left(s_t^h, d_{g,t}, b_t^h, d_t\right) - \theta Q_t^h s_t^h\right] + \lambda_{w,t}^h(\phi d_{g,t} - b_t^h) \\ =& \left((1+\lambda_{r,t}^h)\mathcal{V}_{st} - \lambda_{r,t}^h\theta Q_t^h\right)s_t^h + \left((1+\lambda_{r,t}^h)\mathcal{V}_{gt} + \phi\lambda_{w,t}^h R_{g,t}\right)d_{g,t}/R_{g,t} \\ & - \left((1+\lambda_{r,t}^h)\mathcal{V}_{bt} + \lambda_{w,t}^h\right)b_t^h - \left(1+\lambda_{r,t}^h\right)\mathcal{V}_{dt}d_t. \end{aligned}$$

Eliminating b_t^h by using (A.3) implies that

$$\begin{aligned} \mathbb{L} =& \left[(1+\lambda_{r,t}^h)(\frac{\mathcal{V}_{st}}{Q_t^h} - \mathcal{V}_{bt}) - \theta\lambda_{r,t}^h - \lambda_{w,t}^h\right]Q_t^h s_t^h \\ & + \left((1+\lambda_{r,t}^h)\mathcal{V}_{gt} + \phi\lambda_{w,t}^h R_{g,t} - \left((1+\lambda_{r,t}^h)\mathcal{V}_{bt} + \lambda_{w,t}^h\right)\right)d_{g,t}/R_{g,t} \\ & + \left[(1+\lambda_{r,t}^h)\mathcal{V}_{bt} + \lambda_{w,t}^h\right]n_t^h + \left[(1+\lambda_{r,t}^h)(\mathcal{V}_{bt} - \mathcal{V}_{dt}) + \lambda_{w,t}^h\right]d_t. \end{aligned}$$

Therefore, the first-order condition for s_t^h is given by

$$(1+\lambda_{r,t}^h)(\frac{\mathcal{V}_{st}}{Q_t^h} - \mathcal{V}_{bt}) = \theta\lambda_{r,t}^h + \lambda_{w,t}^h. \tag{A.6}$$

The first-order condition for d_t is given by

$$(1+\bar{\lambda}_{r,t})(\mathcal{V}_{bt}-\mathcal{V}_{dt})+\bar{\lambda}_{w,t}=0. \tag{A.7}$$

This implies that the expected marginal cost of interbank borrowing $(1+\bar{\lambda}_{r,t})\mathcal{V}_{bt}+\bar{\lambda}_{w,t}$ is higher than the expected marginal cost of deposit $(1+\bar{\lambda}_{r,t})\mathcal{V}_{dt}$ if the incentive constraint binds at some states .

The first-order condition for $d_{g,t}$ is given by

$$(1+\bar{\lambda}_{r,t})\mathcal{V}_{gt}+\phi\bar{\lambda}_{w,t}R_{g,t}=(1+\bar{\lambda}_{r,t})\mathcal{V}_{bt}+\bar{\lambda}_{w,t}$$

Or

$$(1+\bar{\lambda}_{r,t})(\mathcal{V}_{bt}-\mathcal{V}_{gt})+\bar{\lambda}_{w,t}=\phi\bar{\lambda}_{w,t}R_{g,t}. \tag{A.8}$$

Comparing (A.7) and (A.8) once again shows that government bonds and deposits are perfect substitutes ($\mathcal{V}_{gt}=\mathcal{V}_{dt}$) if the interbank borrowing constraint is not binding.

With these optimality conditions, the Bellman equation can be rewritten as

$$\begin{aligned}
&V_{t-1}\left(s_{t-1},d_{g,t-1},b_{t-1},d_{t-1}\right)\\
=&\mathbb{E}_{t-1}\Lambda_{t-1,t}\sum_{h=i,n}\pi^h\left\{(1-\sigma)n_t^h+\sigma\left[(1+\lambda_{r,t}^h)\mathcal{V}_{bt}+\lambda_{w,t}^h\right]n_t^h\right\}\\
=&\mathbb{E}_{t-1}\Lambda_{t-1,t}\sum_{h=i,n}\pi^h\Omega_t^h\left[\left[Z_t+(1-\delta)Q_t^h\right]\psi_t s_{t-1}+d_{g,t-1}-R_{bt-1}b_{t-1}-R_{t-1}d_{t-1}\right],
\end{aligned}$$

where

$$\Omega_t^h=\left\{(1-\sigma)+\sigma\left[(1+\lambda_{r,t}^h)\mathcal{V}_{bt}+\lambda_{w,t}^h\right]\right\}.$$

Matching coefficients show that

$$\begin{aligned}
\mathcal{V}_{st}&=\mathbb{E}_t\Lambda_{t,t+1}\sum_{h=i,n}\pi^h\Omega_{t+1}^h\left[Z_{t+1}+(1-\delta)Q_{t+1}^h\right]\psi_{t+1},\\
\mathcal{V}_{gt}&=\mathbb{E}_t\Lambda_{t,t+1}\sum_{h=i,n}\pi^h\Omega_{t+1}^h R_{gt},\\
\mathcal{V}_{bt}&=\mathbb{E}_t\Lambda_{t,t+1}\sum_{h=i,n}\pi^h\Omega_{t+1}^h R_{bt},\\
\mathcal{V}_{dt}&=\mathbb{E}_t\Lambda_{t,t+1}\sum_{h=i,n}\pi^h\Omega_{t+1}^h R_{t}.
\end{aligned}$$

To simplify (A.1), notice that it can be rewritten as

$$\mathcal{V}_{st}s_t^h+\mathcal{V}_{gt}d_{g,t}/R_{g,t}-\mathcal{V}_{bt}b_t^h-\mathcal{V}_{dt}d_t\geq\theta Q_t^h s_t^h.$$

Eliminating b_t^h by using (A.3) yields

$$\left(\theta-(\frac{\mathcal{V}_{st}}{Q_t^h}-\mathcal{V}_{bt})\right)Q_t^h s_t^h\leq\left(\mathcal{V}_{gt}-\mathcal{V}_{bt}\right)d_{g,t}/R_{g,t}+\mathcal{V}_{bt}n_t^h-(\mathcal{V}_{dt}-\mathcal{V}_{bt})d_t.$$

B Equilibrium System

Given the exogenous process of $\{A_t, \psi_t, G_t, D_{gt}\}$, a competitive equilibrium consists of 27 aggregate variables, $\{ Y_t, K_t, C_t, L_t, W_t, I_t, S_t^i, S_t^n, S_t, Q_t^i, Q_t^n, N_t^i, N_t^n, D_t, Z_t, R_t, R_{gt}, R_{bt}, \lambda_{w,t}^i, \lambda_{w,t}^n, \lambda_{r,t}^i, \lambda_{r,t}^n, \mathcal{V}_{st}, \mathcal{V}_{gt}, \mathcal{V}_{bt}, \mathcal{V}_{dt}, T_t \}$, such that the following 27 equations hold:

1. Aggregate output,

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}. \tag{B.1}$$

2. Aggregate capital,

$$K_{t+1} = \psi_{t+1} \left[I_t + (1-\delta) K_t \right]. \tag{B.2}$$

3. Resource constraint,

$$Y_t = C_t + G_t + \left(1 + f\left(\frac{I_t}{I_{t-1}} \right) \right) I_t. \tag{B.3}$$

4. Labor supply,

$$\mathbb{E}_t u_{C_t} W_t = \chi L_t^{\epsilon}, \tag{B.4}$$

with

$$u_{C_t} \equiv (C_t - \gamma C_{t-1})^{-1} - \beta\gamma \left(C_{t+1} - \gamma C_t \right)^{-1}.$$

5. Labor demand,

$$W_t = (1-\alpha) \frac{Y_t}{L_t}. \tag{B.5}$$

6. Tobin's Q,

$$Q_t^i = 1 + f\left(\frac{I_t}{I_{t-1}} \right) + \frac{I_t}{I_{t-1}} f'\left(\frac{I_t}{I_{t-1}} \right) - \mathbb{E}_t \Lambda_{t,t+1} \left(\frac{I_{t+1}}{I_t} \right)^2 f'\left(\frac{I_{t+1}}{I_t} \right). \tag{B.6}$$

7. Supply of equity on investing islands,

$$S_t^i = I_t + (1-\delta) \pi^i K_t. \tag{B.7}$$

8. Supply of equity on non-investing islands,

$$S_t^n = (1-\delta) \pi^n K_t, \tag{B.8}$$

9. The total supply of equity,

$$S_t = S_t^i + S_t^n. \tag{B.9}$$

10. The demand for equity on investing islands,

$$(1+\lambda^i_{r,t})(\frac{\mathcal{V}_{st}}{Q^i_t}-\mathcal{V}_{bt})=\theta\lambda^i_{r,t}+\lambda^i_{w,t}, \tag{B.10}$$

11. The demand for equity on non-investing islands,

$$(1+\lambda^n_{r,t})(\frac{\mathcal{V}_{st}}{Q^n_t}-\mathcal{V}_{bt})=\theta\lambda^n_{r,t}+\lambda^n_{w,t}, \tag{B.11}$$

12. Bank net worth on investing islands,

$$N^i_t=N^i_{o,t}+N^i_{y,t}, \tag{B.12}$$

with

$$N^i_{y,t}=\xi\pi^i\left[Z_t+(1-\delta)Q^i_t\right]\psi_t S_{t-1},$$
$$N^i_{o,t}=\sigma\pi^i\left\{\left[Z_t+(1-\delta)Q^i_t\right]\psi_t S_{t-1}+D_{g,t-1}-R_{t-1}D_{t-1}\right\}.$$

13. Bank net worth on non-investing islands,

$$N^n_t=N^n_{o,t}+N^n_{y,t}, \tag{B.13}$$

with

$$N^n_{y,t}=\xi\pi^n\left[Z_t+(1-\delta)Q^n_t\right]\psi_t S_{t-1},$$
$$N^n_{o,t}=\sigma\pi^n\left\{\left[Z_t+(1-\delta)Q^n_t\right]\psi_t S_{t-1}+D_{g,t-1}-R_{t-1}D_{t-1}\right\}.$$

14. Balance sheet of the banking sector,

$$N^i_t+N^n_t+D_t=D_{g,t}/R_{g,t}+Q^i_tS^i_t+Q^n_tS^n_t. \tag{B.14}$$

15. Capital return,

$$Z_t=\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}A_t^{\frac{1}{\alpha}}W_t^{-\frac{1-\alpha}{\alpha}}. \tag{B.15}$$

16. Euler equation,

$$\mathbb{E}_t\Lambda_{t,t+1}R_t=1, \tag{B.16}$$

with

$$\Lambda_{t,t+1}\equiv\beta\frac{u_{C_{t+1}}}{u_{C_t}}.$$

17. Valuation of government bond,

$$\mathcal{V}_{gt}=\mathbb{E}_t\Lambda_{t,t+1}\sum_{h=i,n}\pi^h\Omega^h_{t+1}R_{gt}, \tag{B.17}$$

with

$$\Omega^h_t=\left\{(1-\sigma)+\sigma\left[(1+\lambda^h_{r,t})\mathcal{V}_{bt}+\lambda^h_{w,t}\right]\right\}.$$

18. Valuation of interbank borrowing,

$$\mathcal{V}_{bt} = \mathbb{E}_t \Lambda_{t,t+1} \sum_{h=i,n} \pi^h \Omega_{t+1}^h R_{bt}. \tag{B.18}$$

19. Interbank borrowing constraint for investing banks,

$$B_t^i \leq \phi \pi^i D_{g,t}, \tag{B.19}$$

with

$$B_t^i = \pi^i D_{g,t}/R_{g,t} + Q_t^i S_t^i - N_t^i - \pi^i D_t.$$

20. Interbank borrowing constraint for non-investing banks,

$$B_t^n \leq \phi \pi^n D_{g,t}, \tag{B.20}$$

with

$$B_t^n = \pi^n D_{g,t}/R_{g,t} + Q_t^n S_t^n - N_t^n - \pi^n D_t.$$

21. Deposit market borrowing constraint for investing banks,

$$\left[\theta - \left(\frac{\mathcal{V}_{st}}{Q_t^i} - \mathcal{V}_{bt}\right)\right] Q_t^i S_t^i \leq \left(\mathcal{V}_{bt} - \mathcal{V}_{dt}\right) \pi^i D_t + \left(\mathcal{V}_{gt} - \mathcal{V}_{bt}\right) \pi^i D_{g,t}/R_{g,t} + \mathcal{V}_{bt} N_t^i. \tag{B.21}$$

22. Deposit market borrowing constraint for non-investing banks,

$$\left[\theta - \left(\frac{\mathcal{V}_{st}}{Q_t^n} - \mathcal{V}_{bt}\right)\right] Q_t^n S_t^n \leq \left(\mathcal{V}_{bt} - \mathcal{V}_{dt}\right) \pi^n D_t + \left(\mathcal{V}_{gt} - \mathcal{V}_{bt}\right) \pi^n D_{g,t}/R_{g,t} + \mathcal{V}_{bt} N_t^n. \tag{B.22}$$

23. Valuation of equity,

$$\mathcal{V}_{st} = E_t \Lambda_{t,t+1} \sum_{h=i,n} \pi^h \Omega_{t+1}^h \left[Z_{t+1} + (1-\delta) Q_{t+1}^h\right] \psi_{t+1}.$$

24. Bank's optimal choice for government bond,

$$(1+\bar{\lambda}_{r,t})\mathcal{V}_{gt} = (1+\bar{\lambda}_{r,t})\mathcal{V}_{bt} + (1-\phi R_{g,t})\bar{\lambda}_{w,t}. \tag{B.23}$$

25. Valuation of deposit,

$$\mathcal{V}_{dt} = E_t \Lambda_{t,t+1} \sum_{h=i,n} \pi^h \Omega_{t+1}^h R_t. \tag{B.24}$$

26. Bank's optimal choice for deposit,

$$(1+\bar{\lambda}_{r,t})(\mathcal{V}_{bt} - \mathcal{V}_{dt}) + \bar{\lambda}_{w,t} = 0. \tag{B.25}$$

27. Government budget constraint,

$$D_{gt}/R_{g,t} - D_{gt-1} = G_t - T_t. \tag{B.26}$$

C Steady State

Assume that the government spending to GDP ratio is exogenously given by $G/Y = g$. The model in the steady state can be summarized as follows:

1. Aggregate output,
$$Y = AK^{\alpha}L^{1-\alpha}. \tag{C.1}$$
2. Aggregate investment,
$$I = \delta K. \tag{C.2}$$
3. Resource constraint,
$$\frac{C}{K} = (1-g)A(\frac{L}{K})^{1-\alpha} - \delta. \tag{C.3}$$
4. Labor supply,
$$\chi L^{\varepsilon} = (1-\alpha)A\left(\frac{K}{L}\right)^{\alpha}\frac{1-\beta\gamma}{1-\gamma}\frac{1}{C}. \tag{C.4}$$
5. Labor demand,
$$W = (1-\alpha)A\left(\frac{K}{L}\right)^{\alpha}. \tag{C.5}$$
6. Tobin's Q,
$$Q^i = 1. \tag{C.6}$$
7. Supply of equity on investing islands,
$$S^i = I + (1-\delta)\pi^i K. \tag{C.7}$$
8. Supply of equity on non-investing islands,
$$S^n = (1-\delta)\pi^n K. \tag{C.8}$$
9. Total supply of equity,
$$S = S^i + S^n. \tag{C.9}$$
10. The demand of equity on investing islands,
$$(1+\lambda^i_{r,t})(\frac{\mathcal{V}_{st}}{Q^i_t} - \mathcal{V}_{bt}) = \theta\lambda^i_{r,t} + \lambda^i_{w,t}. \tag{C.10}$$
11. The demand of equity on non-investing islands,
$$(1+\lambda^n_{r,t})(\frac{\mathcal{V}_{st}}{Q^n_t} - \mathcal{V}_{bt}) = \theta\lambda^n_{r,t} + \lambda^n_{w,t}. \tag{C.11}$$

12. The net worth of banks on investing islands,

$$N^i = \pi^i \left[(\sigma + \xi)(Z + 1 - \delta)K + \sigma D_g - \frac{\sigma}{\beta} D \right]. \tag{C.12}$$

13. The net worth of banks on non-investing islands,

$$N^n = \pi^n \left[(\sigma + \xi) \left[Z + (1 - \delta)Q^n \right] K + \sigma D_g - \frac{\sigma}{\beta} D \right]. \tag{C.13}$$

14. Balance sheet of the banking sector,

$$N^i + N^n + D = D_g/R_g + K + (Q^n - 1)\pi^n(1 - \delta)K. \tag{C.14}$$

15. Capital return,

$$Z = \alpha A \left(\frac{L}{K} \right)^{1-\alpha}. \tag{C.15}$$

16. Euler equation,

$$R = \frac{1}{\beta}. \tag{C.16}$$

17. Valuation of government bond,

$$\mathcal{V}_g = \beta \left\{ (1 - \sigma) + \sigma \left[(1 + \bar{\lambda}_r)\mathcal{V}_b + \bar{\lambda}_w \right] \right\} R_g. \tag{C.17}$$

18. Valuation of interbank borrowing,

$$\mathcal{V}_b = \beta \left\{ (1 - \sigma) + \sigma \left[(1 + \bar{\lambda}_r)\mathcal{V}_b + \bar{\lambda}_w \right] \right\} R_b. \tag{C.18}$$

19. Interbank borrowing constraint for investing banks,

$$(\frac{1}{R_g} - \phi)\pi^i D_g + Q^i S^i \leq N^i + \pi^i D. \tag{C.19}$$

with equility holds if $\lambda_w^h > 0$ while strict inequility implies $\lambda_w^h = 0$

20. Interbank borrowing constraint for non-investing banks,

$$(\frac{1}{R_g} - \phi)\pi^n D_g + Q^n S^n \leq N^n + \pi^n D. \tag{C.20}$$

21. Deposit market borrowing constraint for investing banks,

$$\left[\theta - \left(\frac{\mathcal{V}_{st}}{Q_t^i} - \mathcal{V}_{bt} \right) \right] Q_t^i S_t^i \leq (\mathcal{V}_{bt} - \mathcal{V}_{dt})\, \pi^i D_t + (\mathcal{V}_{gt} - \mathcal{V}_{bt})\, \pi^i D_{g,t}/R_{g,t} + \mathcal{V}_{bt} N_t^i, \tag{C.21}$$

where equality holds if $\lambda^i > 0$ while strict inequality implies $\lambda^i = 0$.

22. Deposit market borrowing constraint for non-investing banks,

$$\left[\theta - \left(\frac{\mathcal{V}_{st}}{Q_t^n} - \mathcal{V}_{bt}\right)\right] Q_t^n S_t^n \leq \left(\mathcal{V}_{bt} - \mathcal{V}_{dt}\right) \pi^n D_t + \left(\mathcal{V}_{gt} - \mathcal{V}_{bt}\right) \pi^n D_{g,t}/R_{g,t} + \mathcal{V}_{bt} N_t^n, \tag{C.22}$$

where equality holds if $\lambda^n > 0$ while strict inequality implies $\lambda^n = 0$.

23. Valuation of equity,

$$\begin{aligned}\mathcal{V}_s =& \beta\pi^i \left\{(1-\sigma) + \sigma\left[(1+\lambda_r^i)\mathcal{V}_b + \lambda_w^i\right]\right\}[Z + (1-\delta)] \\ &+ \beta\pi^n \left\{(1-\sigma) + \sigma\left[(1+\lambda_r^n)\mathcal{V}_b + \lambda_w^n\right]\right\}[Z + (1-\delta)Q^n]. \end{aligned} \tag{C.23}$$

24. Bank's optimal choice for deposit,

$$(1+\bar{\lambda}_r)(\mathcal{V}_b - \mathcal{V}_g) + \bar{\lambda}_w = \phi\bar{\lambda}_w R_g. \tag{C.24}$$

25. Valuation of deposit,

$$\mathcal{V}_d = \beta\left\{(1-\sigma) + \sigma\left[(1+\bar{\lambda}_r)\mathcal{V}_b + \bar{\lambda}_w\right]\right\} R. \tag{C.25}$$

26. Bank's optimal choice for deposit,

$$(1+\bar{\lambda}_r)(\mathcal{V}_b - \mathcal{V}_d) + \bar{\lambda}_w = 0. \tag{C.26}$$

27. Government budget constraint,

$$D_g(1/R_g - 1) = G - T. \tag{C.27}$$

The model is block-recursive in the sense that the banking sector impacts the real economy only through the capital return (C.15). For any given value of capital return Z, combining (C.3) and (C.4) yields the steady state value of K, L and C. Then using (C.1), (C.2), (C.5), (C.7), (C.8), (C.9), the steady state value of Y, I, W, S^i, S^n, and S can be pinned down. To pin down the capital return Z, one needs to combine with the rest of the system taking into the account of financial frictions.

D Proof of Proposition 1

With the assumption that (10) is not binding, I obtain $\lambda_{r,t}^i = \lambda_{r,t}^n = 0$. Using (C.12) and (C.13), the balance sheet condition (C.14) for the banking sector can be rewritten as

$$\begin{aligned}\frac{D_g}{K}(1/R_g - \sigma) - \left(1 - \frac{\sigma}{\beta}\right)\frac{D}{K} =& (\sigma+\xi)(Z + 1 - \delta) \\ &+ \pi^n(\sigma + \xi - 1)(1-\delta)(Q^n - 1) - 1. \end{aligned} \tag{D.1}$$

Eliminating N^i and S^i in (C.19) by using (C.7) and (C.12), I obtain

$$\pi^i\left[\frac{1}{R_g}-\phi-\sigma\right]\frac{D_g}{K}\leq\left[\pi^i(\sigma+\xi)(Z+1-\delta)-\delta-(1-\delta)\pi^i\right]+\pi^i(1-\frac{\sigma}{\beta})\frac{D}{K}.$$

Using (D.1) to eliminate D/K in the above condition yields

$$\delta+\pi^i(1-\delta)(Q^n-1)(\sigma+\xi-1)\leq\phi\frac{\pi^i}{\pi^n}\frac{D_g}{K}. \tag{D.2}$$

Notice that if $\phi=0$, I have the case where there is no interbank lending at all. Under the calibration of Gertler and Kiyotaki (2010), i.e. $\sigma+\xi-1<0$, the resulting capital price Q^n on non-investing islands will be inefficiently large. Intuitively, with borrowing constraint, non-investing banks are not willing to lend out their idle money; instead, non-investing banks use the idle money to purchase existing equities, which only pushes up the asset prices.

Recalling that $\lambda^i_{r,t}=\lambda^n_{r,t}=0$ by assumption, equations (C.26) and (C.24) can be rewritten as

$$\mathcal{V}_d-\mathcal{V}_b=\bar{\lambda}_w, \tag{D.3}$$

$$\mathcal{V}_g-\mathcal{V}_b=(1-\phi R_g)\bar{\lambda}_w, \tag{D.4}$$

respectively.

Conjecture that $\lambda^i_w>0$ and $\lambda^n_w=0$, i.e. only the inter-bank borrowing constraint on the investing islands is binding. As a result, from (C.10) and (C.11) I obtain

$$\frac{\mathcal{V}_s}{Q^i}-\mathcal{V}_b=\frac{\bar{\lambda}_w}{\pi^i},\ Q^i=1, \tag{D.5}$$

and

$$\frac{\mathcal{V}_s}{Q^n}-\mathcal{V}_b=0. \tag{D.6}$$

Using (D.3) to eliminate $\mathcal{V}_d$ in (C.25), I obtain

$$\mathcal{V}_b=1-\bar{\lambda}_w.$$

Combining with (D.3), (D.4), and (D.5) yields

$$\mathcal{V}_d=1,$$
$$\mathcal{V}_g=1-\phi R_g\bar{\lambda}_w,$$
$$\mathcal{V}_s=1+\frac{\pi^n}{\pi^i}\bar{\lambda}_w.$$

It follows from (D.6) that

$$Q^n=\frac{\mathcal{V}_s}{\mathcal{V}_b}=\frac{1+\frac{\pi^n}{\pi^i}\bar{\lambda}_w}{1-\bar{\lambda}_w}. \tag{D.7}$$

I now turn to the solution of capital return Z. It follows from (C.23) that

$$\begin{aligned}\mathcal{V}_s =& \beta\left\{(1-\sigma)+\sigma\left[\mathcal{V}_b+\bar{\lambda}_w\right]\right\}[Z+(1-\delta)] \\ &+\beta\pi^n\left[(1-\sigma)+\sigma\mathcal{V}_b\right][(1-\delta)(Q^n-1)]\,.\end{aligned}$$

Combining with the conditions of $\mathcal{V}_s$, $\mathcal{V}_b$ and Q^n, I obtain

$$Z+(1-\delta)=\frac{1}{\beta}\left[1+\frac{\pi^i}{\pi^n}\bar{\lambda}_w\left(1-\beta\,(1-\delta)\left(\frac{1-\sigma}{1-\bar{\lambda}_w}+\sigma\right)\right)\right]. \tag{D.8}$$

This suggests that a wedge between the capital return rate and the deposit rate will arise if the wholesale interbank market is under pressure of borrowing constraint such that $\bar{\lambda}_w \neq 0$. As shown by Appendix C, financial frictions affect the real block only through the capital return Z, which in turn depends on $\bar{\lambda}_w$. Therefore, the steady steate of the model can be summarized by two variables $\bar{\lambda}_w$ and Q^n with two equations (D.2) and (D.7).

Suppose that the interbank market is under pressure of financial friction such that $\bar{\lambda}_w > 0$ and (D.2) is binding. Substituting (D.7) into the incentive constraint (D.2) yields

$$\bar{\lambda}_w=\frac{\delta-\frac{\pi^i}{\pi^n}\phi\frac{D_g}{K}}{\delta-\frac{\pi^i}{\pi^n}\phi\frac{D_g}{K}+(1-\delta)(1-\sigma-\xi)}. \tag{D.9}$$

It is straightforward that $\bar{\lambda}_w$ is decreasing in D_g/K if $1-\sigma-\xi>0$. It follows that with $D_g/K \in \left[0, \frac{\pi^n}{\pi^i}\frac{\delta}{\phi}\right]$, I have

$$0\le\bar{\lambda}_w\le\frac{\delta}{\delta+(1-\delta)(1-\sigma-\xi)}\equiv\bar{\lambda}_w^{**}\le 1,$$

where $\bar{\lambda}_w^{**}$ is defined as the upper bound of the Lagrangian multiplier. On the other hand, if $D_g/K > \frac{\pi^n}{\pi^i}\frac{\delta}{\phi}$, the the interbank borrowing constraint is not binding, and $\bar{\lambda}_w = 0$. Using (D.7) and (D.8), one can easily solve for other endogenous variables.

E Proof of Proposition 2

Equation (C.16) shows that the deposit rate $R = 1/\beta$. Taking the ratio of (C.17) over (C.25) yields

$$R_g=\frac{1}{\beta}(1-\phi R_g\bar{\lambda}_w). \tag{E.1}$$

It follows that

$$R_g=\frac{1}{\beta+\phi\bar{\lambda}_w}.$$

Combining with (C.17) and (C.18) yields

$$R_b=\frac{1}{\beta}(1-\bar{\lambda}_w).$$

To see the connection between D_g/K and $\bar{\lambda}_W$, recall that Q^n is monotonically increasing in $\bar{\lambda}_w$ by (D.7). Given the assumption that $1-\sigma-\xi>0$, the left-hand side of the borrowing constraint

(D.2) is decreasing in $\bar{\lambda}_w$. Then one can show that the interbank borrowing constraint is binding and $\bar{\lambda}^w > 0$ if $\phi\frac{D_g}{K} < \frac{\pi^n}{\pi^i}\delta$; on the other hand, if $\phi\frac{D_g}{K} \geq \frac{\pi^n}{\pi^i}\delta$, then the interbank borrowing constraint is not binding and $\bar{\lambda}_w = 0$.

Condition (D.8) implicitly defines a function $Z = Z(\bar{\lambda}_w)$. Taking derivative shows

$$\frac{\partial Z}{\partial \bar{\lambda}_w} = \beta\frac{\pi^i}{\pi^n}\left[1-\beta(1-\delta)\sigma - \frac{\beta(1-\delta)(1-\sigma)}{(1-\bar{\lambda}_w)^2}\right],$$

which is greater than 0 at $\bar{\lambda}_w = 0$ and decreasing in $\bar{\lambda}_w \in \left[0, \bar{\lambda}_w^{**}\right]$. Imposing $\partial Z/\partial\bar{\lambda}_w = 0$, I obtain[9]

$$\bar{\lambda}_w^* = 1 - \sqrt{\frac{\beta(1-\delta)(1-\sigma)}{1-\beta(1-\delta)\sigma}}.$$

Therefore, if $\bar{\lambda}_w^* < \bar{\lambda}_w^{**}$, the capital return $Z + 1 - \delta$ increasing in $\bar{\lambda}_w$ if $\bar{\lambda}_w \in \left[0, \bar{\lambda}_w^*\right]$, decreasing in $\bar{\lambda}_w$ if $\bar{\lambda}_w \in \left[\bar{\lambda}_w^*, \bar{\lambda}_w^{**}\right]$. On the other hand, if $\bar{\lambda}^* > \bar{\lambda}^{**}$, then the capital return $Z + 1 - \delta$ is increasing in $\left[0, \bar{\lambda}_w^{**}\right]$. Noting that $Z(0) + 1 - \delta = 1/\beta$, the necessary and sufficient condition of $Z(\bar{\lambda}_w) + 1 - \delta > 1/\beta$ for $\bar{\lambda}_w > 0$ is to ensure $Z(\bar{\lambda}_w^{**}) + 1 - \delta > 1/\beta$. This is equivalent to

$$\frac{\xi}{1-\sigma} < \frac{1-\beta(1-\sigma)-\delta}{(1-\delta)(1-\beta(1-\sigma))}.$$

It turns out that the above condition is satisfied under the parametrization of Gertler and Kiyotaki (2010). Noting that $Z(\bar{\lambda}_w)$ is an increasing function when $\bar{\lambda}_w$ is small and that $Z(0)+1-\delta = 1/\beta$, it follows that $Z + 1 - \delta > R$ and $Z(\bar{\lambda}_w)$ is decreasing in D_g/K when $\bar{\lambda}_w$ is small.

I will focus on the case where $\bar{\lambda}_w \in \left[0, \min\{\bar{\lambda}_w^*, \bar{\lambda}_w^*\}\right]$. This means that there is shortage of public debt, but not so much such that the speculation channel dominates the credit supply channel. As discussed above, it follows that Z is decreasing in D_g/K. By (C.15) I obtain

$$\frac{D_g}{K} = \frac{D_g}{Y}\frac{Y}{K} = \frac{1}{\alpha}Z\frac{D_g}{Y}.$$

This implies that D_g/Y is monotically increasing in D_g/Y. Therefore, instead of studying the conventional debt-to-gdp ratio D_g/Y, I can equivalently focus on debt-to-capital ratio D_g/K.

F Proof of Proposition 3

Suppose that labor supply is fixed. Recalling (C.15), this assumption suggests that aggregate output can be derived by using the following condition:

$$Y = A(\frac{\alpha A}{Z})^{\frac{\alpha}{1-\alpha}}L.$$

Due to financial frictions in the interbank market, the ways of financing government spending have different impacts on the capital return Z and output Y. Consider an increase in government

[9] The other solution is greater than 1, and hence omitted.

spending which is all financed by an increase in lump-sum tax while the public debt level D_g is fixed. Conjecture that the capital level K is unchanged. Then by (D.9) and (D.8), both of $\bar{\lambda}_w$ and Z are also unaffected. Since the labor supply is exogenous, (C.15) implies that capital is still at its original value, which verifies the initial conjecture that the capital level K is unchanged. This suggests that if government spending is financed by lump-sum tax, it has no impacts on capital return and output.

Now suppose that the government spending is financed by an increase in public debt. This must be followed by an increase in capital K. Suppose not. Then condition (D.9) shows that $\bar{\lambda}_w$ is smaller. With $\bar{\lambda}_w \in \left[0, \min\{\bar{\lambda}^*_w, \bar{\lambda}^*_w\}\right]$, Appendix E suggests that Z is increasing in $\bar{\lambda}_w$. Therefore, an increase in public debt leads to lower Z, and hence higher capital K. This contradicts with the conjecture that capital K is unchanged or lower. Thus, an increase in public debt leads to higher capital accumulation K, lower capital return Z and higher output Y.

G Proof of Proposition 4

Suppose that all the borrowing constraints are not binding so that $\lambda^i_r = \lambda^n_r = \lambda^i_w = \lambda^n_w = 0$. Then by (C.6), (C.10), and (C.10), I obtain $Q^n = Q^i = 1$ and $V_s = V_b$. Combining with (C.16), (C.17), (C.18), (C.23), (C.24), (C.25), and (C.26) yields $R_g = R_b = Z + 1 - \delta = R = 1/\beta$ and $\mathcal{V}_s = \mathcal{V}_b = \mathcal{V}_g = \mathcal{V}_d = 1$. Using (C.12), (C.13), (C.14) to eliminate N^i, N^n, D in (C.19) and (C.21) yield the critical values of D_g/K and θ such that (C.19) and (C.21) are not binding. With these critical values, one can verify that (C.20) and (C.22) are not binding as well.